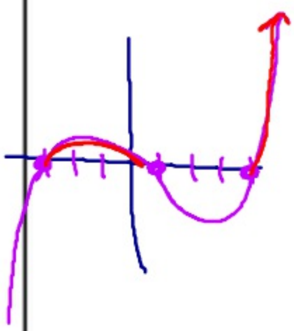
2.8: Solving Inequalities in One Variable

Determine the x-values that cause the polynomial function to be a) zero, b) positive, and c) negative



a) $f(x) = (x + 3)(x - 1)(x - 4)$

a) $f(x) = 0$
$x = -3 \quad x = 1 \quad x = 4$

b) $f(x) > 0$
$(-3, 1) \cup (4, \infty)$

c) $f(x) < 0$
$(-\infty, -3) \cup (1, 4)$

-3 1 4

$f(-4) = (-1)(-5)(-8) = -40$

b) $f(x) = (x^2 + 3)(x + 1)(x - 2)$

$x^2 + 3 = 0$
$-3 \ -3$
$x^2 = -3$
$x = \pm\sqrt{-3}$

a) $f(x) = 0$
$x = -1 \quad x = 2$

b) $f(x) > 0$
$(-\infty, -1) \cup (2, \infty)$

c) $f(x) < 0$
$(-1, 2)$

\+ above | − below | + above
-1 2

$f(-2) = (7)(-1)(-4) = 28$
$f(0) = (3)(1)(-2) = -6$
$f(3) = (12)(4)(1) = 48$

c) $f(x) = (x + 3)^3(x^2 + 1)(x - 4)^2$

a) $f(x) = 0$
$x = -3 \quad x = 4$

b) $f(x) > 0$
$(-3, 4) \cup (4, \infty)$ ← ~~$(-3, \infty)$?~~

c) $f(x) < 0$
$(-\infty, -3)$

below | above | above
-3 4

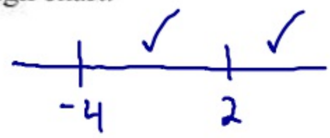
$f(-4) = (-1)(17)(64) < 0$
$f(0) = (+)(+)(+) > 0$
$f(5) = (+)(+)(+) > 0$

Solve the polynomial inequality using a sign chart.

a) $(x-2)^2(x+4) > 0$

greater/above

$f(x) > 0$



$f(-5) = (+)(-1) < 0$

$f(0) = (+)(4) > 0$

$f(3) = (+)(7) > 0$

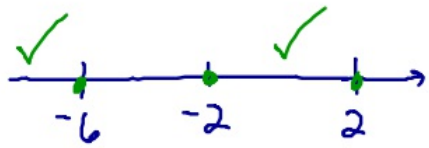
Zeros: $x = 2$

$x = -4$

$(-4, 2) \cup (2, \infty)$

b) $(x+2)(x^2+4x-12) \leq 0$

$(x+2)(x+6)(x-2) \leq 0$



$x = -2 \quad x = -6 \quad x = 2$

$(-\infty, -6] \cup [-2, 2]$

$f(-7) = (-)(-)(-) \leq 0$

$f(-3) = (-)(+)(-) > 0$

$f(0) = (+)(+)(-) \leq 0$

$f(3) = (+)(+)(+) > 0$

* c) $2x^3 - 7x^2 - 10x + 24 < 0$

q (2), p (24)

$p = 24$

$\pm1, \pm2, \pm3, \pm4,$

$\pm6, \pm8, \pm12, \pm24$

$q = \pm1, \pm2$

$\frac{p}{q} = \pm1, \pm2, \pm3, \pm4$

$\pm6, \pm8, \pm12, \pm24$

$\pm\frac{1}{2}, \pm\frac{3}{2}$

-2	2	-7	-10	24
		-4	22	-24
	2	-11	12	0

	Solve the polynomial inequality graphically. 13) $x^3 - x^2 - 2x \geq 0$ 14) $3x^4 - 5x^3 - 12x^2 + 12x + 16 < 0$ Determine the real values of x that cause the function to be a) zero, b) undefined, c) positive, and d) negative A) $f(x) = \frac{x-4}{(3x+2)(x+3)}$ B) $f(x) = \frac{x-3}{(x+2)\sqrt{x+3}}$ C) $f(x) = \frac{\sqrt{x-3}}{(x+2)(x-5)}$

When writing intervals use brackets when necessary on zero's and parenthesis always on VA	Solve the inequality A) $\frac{x-2}{x^2-9}<0$ B) $\frac{x^2-9}{x^2+9}\geq 0$ C) $\frac{x^2-x-2}{x^2+8x-9}\leq 0$ D) $\frac{x^3-9x}{x^2+4}>0$

Solve the Inequality

A) $(x-3)\sqrt{x+1} \geq 0$

B) $\dfrac{x^3(x-4)}{(x+1)^2} < 0$

	C) $x^2 + \frac{9}{x} \geq 0$ D) $\frac{5}{x+3} + \frac{3}{x-1} < 0$

Summary of the Characteristics of Graphs of Rational Functions

Horizontal Asymptote

- If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote
  - Check for an oblique/slant asymptote

  $$f(x)=\frac{x^2+2x+1}{x+3}$$
- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is y = 0

  $$f(x)=\frac{3x+1}{x^2+3x+1}$$
- If the degree of the numerator and denominator are the same, the horizontal asymptote will be y = the coefficients of the highest power divided by each other

  $$f(x)=\frac{5x^2+2x+1}{2x^2+3}$$

Vertical Asymptote

- Set the denominator equal to 0
- Make sure the values you get are asymptotes and not holes
- Simplify the original function by factoring
  - Whatever does not cancel is a vertical asymptote

$$f(x)=\frac{x^2+4x+3}{x+5}=\frac{(x+3)(x+1)}{(x+5)}=$$

Holes

- Set the denominator equal to zero
- Make sure the values you get are holes and not vertical asymptotes
- Simplify the original function by factoring
  - Whatever cancels is a hole

$$f(x)=\frac{x^2+4x+3}{x+1}=\frac{(x+3)(x+1)}{(x+1)}=(x+3)$$

Oblique/Slant Asymptote

- If the degree of the numerator is greater than the degree of the denominator, use long division or synthetic division to find the asymptote (y =   )
  - Ignore the remainder
  - If there is no remainder then there is not a slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

X-intercept

- Set the function = 0 (y = 0)
  - You only need to worry about the numerator in a rational function

Y-intercept

- Set the x = 0
  - The values without x will give you your y-intercept