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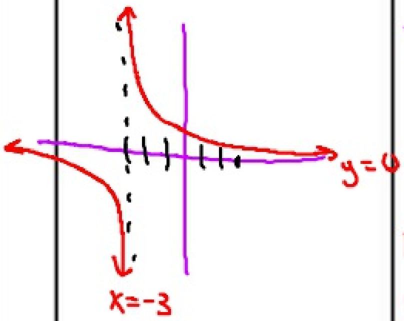
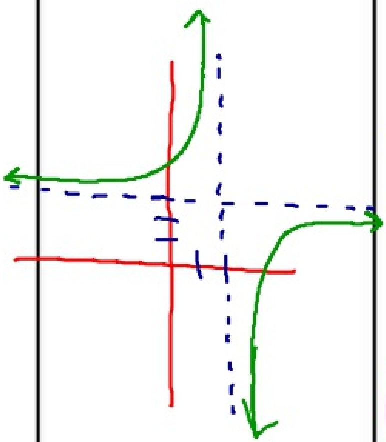
$f(x) = \frac{1}{x}$





Describe how the graph of the given function can be obtained by transforming the graph of the reciprocal/inverse function. Identify the horizontal and vertical asymptotes and use limits to describe the corresponding behavior. Sketch the graph of the function.

a) $\boxed{f(x) = \frac{2}{x+3}}$ Left 3, Vertical Stretch factor of 2 $\quad f(x) = 2\left(\frac{1}{x+3}\right)$

V.A. $x+3=0$
$x=-3$

$\lim_{x \to -3^-} f(x) = -\infty \qquad \lim_{x \to -3^+} f(x) = \infty$

$x=-3.1 \quad y = \frac{2}{-3.1+3}$ $\quad y = \frac{2}{-.1} = -20$

$x=-2.9 \quad y = \frac{2}{-2.9+3}$ $\quad y = \frac{2}{.1} = 20$

HA: $y=0$

because the power on bottom is bigger

$\boxed{\lim_{x \to \pm\infty} f(x) = 0}$

b) $f(x) = \frac{3x-7}{x-2}$

$x=1.9 \quad y = \frac{3(1.9)-7}{1.9-2} = \frac{neg}{neg}$

$x=2.1 \quad y = \frac{3(2.1)-7}{2.1-2} = \frac{neg}{pos}$

VA: $x=2$ $\longrightarrow$ $\lim_{x \to 2^-} f(x) = +\infty \qquad \lim_{x \to 2^+} f(x) = -\infty$

HA: $y = \frac{3}{1} = 3$ $\qquad \lim_{x \to \pm\infty} f(x) = 3$

$f(x) = \frac{3x-7}{x-2}$

right 2

up 3

reflect over x-axis

$$\begin{array}{r|rr} 2 & 3 & -7 \\ & & 6 \\ \hline & 3 & \boxed{-1} \end{array}$$

$f(x) = \frac{3x-7}{x-2} = 3 + \frac{-1}{x-2}$

Find the horizontal and vertical asymptotes of f(x). Use limits to describe the corresponding behavior.

a) $f(x)=\dfrac{5x^2}{x^2+2}$

VA: $x^2+2=0$

$x^2 \neq -2$

NO VA

HA: $y=5$ $\quad \lim_{x\to\pm\infty} f(x)=5$

$y=\dfrac{-1.9+5}{(-1.9)^2+2(-1.9)}$

b) $f(x)=\dfrac{x+5}{x^2+2x}$

HA: $y=0$

$\lim_{x\to\pm\infty} f(x)=0$

V.A: $x^2+2x=0$

$x(x+2)=0$

$x=0 \qquad x+2=0$

$x=-2$

$\lim_{x\to -2^-} f(x)=\infty \qquad \lim_{x\to -2^+} f(x)=-\infty$

$x=-2.1 \quad y=\dfrac{-2.1+5}{(-2.1)^2+2(-2.1)}=\dfrac{+}{+} \qquad x=-1.9$

$\lim_{x\to 0^-} f(x)=-\infty \qquad \lim_{x\to 0^+} f(x)=+\infty$

$x=-.1 \quad y=\dfrac{+}{(-.1)^2+2(-.1)} \qquad y=\dfrac{5.1}{.1^2+2(.1)}$

Find the end behavior asymptote

a) $f(x)=\dfrac{2x^2+2x-3}{x+3} = 2x-4+\dfrac{9}{x+3}$ $\qquad$ b) $f(x)=\dfrac{x^3+1}{x-1}$

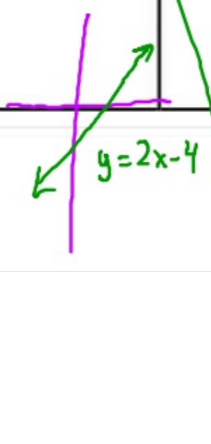
HA: None
power on top
is greater than
power on bottom
- End behavior
approaches $+\infty$
or $-\infty$

$$\begin{array}{r|rrr} -3 & 2 & 2 & -3 \\ & & -6 & 12 \\ \hline & 2 & -4 & \boxed{9} \end{array}$$

End Behavior asy

End Behavior Asy: $y=2x-4$ (Slant asymptote)

$\lim_{x\to\infty} 2x-4=\infty \qquad \lim_{x\to -\infty} 2x-4=-\infty$



b) $f(x) = \dfrac{x^3+1}{x-1} = x^2 + x + 1 + \dfrac{2}{x-1}$

$$\begin{array}{c|cccc} 1 & 1 & 0 & 0 & 1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{2} \end{array}$$

$$\boxed{\begin{array}{l} \lim\limits_{x \to -\infty} x^2+x+1 = \infty \\ \lim\limits_{x \to \infty} x^2+x+1 = \infty \end{array}}$$

Summary of the Characteristics of Graphs of Rational Functions

Horizontal Asymptote

- If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote
  - Check for an oblique/slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is y = 0

$$f(x)=\frac{3x+1}{x^2+3x+1}$$

- If the degree of the numerator and denominator are the same, the horizontal asymptote will be y = the coefficients of the highest power divided by each other

$$f(x)=\frac{5x^2+2x+1}{2x^2+3}$$

Vertical Asymptote

- Set the denominator equal to 0
- Make sure the values you get are asymptotes and not holes
- Simplify the original function by factoring
  - Whatever does not cancel is a vertical asymptote

$$f(x)=\frac{x^2+4x+3}{x+5}=\frac{(x+3)(x+1)}{(x+5)}=$$

Holes

- Set the denominator equal to zero
- Make sure the values you get are holes and not vertical asymptotes
- Simplify the original function by factoring
  - Whatever cancels is a hole

$$f(x)=\frac{x^2+4x+3}{x+1}=\frac{(x+3)(x+1)}{(x+1)}=(x+3)$$

Oblique/Slant Asymptote

- If the degree of the numerator is greater than the degree of the denominator, use long division or synthetic division to find the asymptote (y =   )
  - Ignore the remainder
  - If there is no remainder then there is not a slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

X-intercept

- Set the function = 0 (y = 0)
  - You only need to worry about the numerator in a rational function

Y-intercept

- Set the x = 0
  - The values without x will give you your y-intercept