

Construct the first 3 nonzero terms and the general term of the Maclaurin Series generated by the function and give the interval of convergence.

A) $f(x) = e^{5x} = 1 + 5x + \frac{(5x)^2}{2}$ general term = $\frac{(5x)^n}{n!}$

I.O.C
 $-\infty < x < \infty$

$e^x = 1 + x + \frac{x^2}{2}$
 general term = $\frac{x^n}{n!}$

I.O.C
 $-1 < 2x \leq 1$
 $-\frac{1}{2} < x \leq \frac{1}{2}$

B) $f(x) = \ln(1+2x) = (2x) - \frac{(2x)^2}{2} + \frac{(2x)^3}{3}$ general = $\frac{(-1)^{n-1} (2x)^n}{n}$

I.O.C
 $-1 < x \leq 1$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$
 general = $(-1)^{n-1} \frac{x^n}{n}$

$f(x) = \frac{1}{1+(x+4)} \rightarrow 1 - (x+4) + (x+4)^2 - \dots + (x+4)^n$

geometric
 $\frac{a}{1-r}$

C) $f(x) = \frac{1}{x+5} = \frac{1}{5+x} = \frac{\frac{1}{5}}{\frac{5}{5} + \frac{x}{5}} = \frac{\frac{1}{5}}{1 + \frac{x}{5}}$

First term = $\frac{1}{5}$

common ratio = $-\frac{x}{5}$

3 terms = $\frac{1}{5} - \frac{x}{25} + \frac{x^2}{125}$

general term = $(-1)^n \left(\frac{1}{5}\right) \left(\frac{x}{5}\right)^n = \frac{1}{5} \left(\frac{-x}{5}\right)^n$

$$1 + \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \dots$$

$$1 + 5 + \frac{5^2}{2} + \frac{5^3}{3!} + \frac{5^4}{4!} + \dots$$

Determine which value the series converges to. (Determine the value of the series/Determine the sum of the series)

$$A) \sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$$

geometric
1st term = 1
 $r = \frac{4}{5}$

$$\text{Sum} = f(x) = \frac{1}{1 - \frac{4}{5}} = \frac{1}{\left(\frac{1}{5}\right)} = 5$$

$$B) \sum_{n=0}^{\infty} \frac{5^n}{n!} = e^5$$

$$e^x = \sum \frac{x^n}{n!} \quad x=5$$

$$C) \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{4}\right)^{2n+1}}{(2n+1)!}$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$D) \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{2}\right)^{2n+1}}{2n+1}$$

$$\arctan\left(\frac{1}{2}\right)$$

$2n+1 \rightarrow$ odds
 $2n \rightarrow$ evens
 $n \rightarrow$ all #'s