

6. Given the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} =$$

b. Use the Ratio Test to determine the Interval of Convergence of the series.

c. Enter the first 6 terms into y_1 of your calculator. Use $X[-3,3]_1$ and $Y[0,10]$ as your window.

d. What function does it look like the series represents? That function is the sum of this series.

f. Take the derivative of $\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots = e^x$

g. Take the anti-derivative of $\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots = e^x$

7. Given the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} =$$

b. Use the Ratio Test to determine the Interval of Convergence of the series. Don't forget to test your endpoints.

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{n} \cdot \frac{n}{(-1)^{n-1} x^n} \right| = |x| < 1 \quad \boxed{-1 < x \leq 1}$$

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (-1)^n}{n} = \frac{(-1)^{2n+1}}{n}$
 $x = 1 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

c. Enter the first 6 terms into y_1 of your calculator. Use your interval of convergence for your x window and Y[-2,2] as your window.

d. What function does it look like the series represents? That function is the sum of this series.

f. Take the derivative of $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + \dots = \ln(1+x)$

g. Compare your derivative in part f to the series you wrote in problem 2a.

h. Take the anti-derivative of

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + \dots = \ln(1+x)$$

8. Given the series $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} =$$

b. Use the Ratio Test to determine the Interval of Convergence of the series. Don't forget to test your endpoints.

c. Enter the first 6 terms into y_1 of your calculator. Use your interval of convergence for your x window and $Y\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ as your window.

d. What function does it look like the series represents? That function is the sum of this series.

f. Take the derivative of

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + \dots = \arctan(x)$$

g. Compare your derivative in part f to the series you wrote in problem 3a.

h. Take the anti-derivative of $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + \dots =$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

1. Find the first 4 terms of the series
2. Write the rule for the series
3. Find the interval of convergence
4. Take the derivative of the function and series
5. Take the anti-derivative of the function and the series

5) $f(x) = x^2 e^{x^3}$

replace x with x^3

$$e^{x^3} = 1 + (x^3) + \frac{(x^3)^2}{2!} + \frac{(x^3)^3}{3!} + \dots + \frac{(x^3)^n}{n!} =$$

$$e^{x^3} = 1 + x^3 + \frac{x^6}{2!} + \frac{x^9}{3!} + \dots + \frac{(x^3)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{3n}}{n!}$$

$$f(x) = x^2 e^{x^3} = x^2 + x^5 + \frac{x^8}{2!} + \frac{x^{11}}{3!} + \dots + \frac{x^{3n+2}}{n!}$$

6) $f(x) = x e^{x^4}$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots \frac{(-1)^n x^{2n+1}}{(2n+1)}$$

10n+5

1. Find the first 4 terms of the series
2. Write the rule for the series
3. Find the interval of convergence
4. Take the derivative of the function and series
5. Take the anti-derivative of the function and the series

7) $f(x) = x^2 \tan^{-1}(x^5)$

$$\arctan(x^5) = x^5 - \frac{x^{15}}{3} + \frac{x^{25}}{5} + \dots \frac{(-1)^n x^{5(2n+1)}}{(2n+1)}$$

$$f(x) = x^2 \arctan(x^5) = x^7 - \frac{x^{17}}{3} + \frac{x^{27}}{5} + \dots \frac{(-1)^n x^{10n+7}}{(2n+1)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \frac{(-1)^{n-1} x^n}{n}$$

8) $f(x) = \ln(1-x^4)$

$$f(x) = (-x^4) - \frac{(-x^4)^2}{2} + \frac{(-x^4)^3}{3} - \frac{(-x^4)^4}{4} + \dots \frac{(-1)^{n-1} (-x^4)^n}{n}$$

$$= -x^4 - \frac{x^8}{2} - \frac{x^{12}}{3} - \frac{x^{16}}{4} + \dots \frac{(-1)^{n-1} (-x^{4n})}{n}$$

Construct the first 3 nonzero terms and the general term of the Maclaurin Series generated by the function and give the interval of convergence.

A) $f(x) = e^{5x}$

B) $f(x) = \ln(1+2x)$

C) $f(x) = \frac{1}{x+5}$