$\left(\frac{1}{1}\right)^1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^3+\left(\frac{1}{4}\right)^4 \cdots \leftarrow \frac{1}{1^1}+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\cdots$

What you'll Learn About

Root Test

A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{10^n}$ converges Absolutely

$r = -\frac{1}{10}$

$|r| < 1$

B) $\sum_{n=1}^{\infty} \frac{(1)}{n^n}$ Converges

$\lim_{n\to\infty} \left|\sqrt[n]{\frac{1}{n^n}}\right| =$

$\lim_{n\to\infty} \left(\frac{1}{n^n}\right)^{1/n} = \frac{1^{1/n}}{(n^n)^{1/n}} = \frac{1}{n} = 0 < 1$

The bases are approaching a number smaller than 1 so the series converges

C) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+10}\right)^n$

$\lim_{n\to\infty} \left[\left(\frac{n}{3n+10}\right)^n\right]^{1/n} = \frac{1}{3} < 1$

converges

D) $\sum_{n=1}^{\infty} \left(\frac{5n}{3n+10}\right)^{2n}$

$\lim_{n\to\infty} \left[\left(\frac{5n}{3n+10}\right)^{2n}\right]^{1/n} = \left(\frac{5}{3}\right)^2 > 1$

diverges

E) $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+10}\right)^n$ diverges

$\lim_{n\to\infty} \left|\left(\frac{-2n}{n+10}\right)^n\right|^{1/n} = 2 > 1$

What you'll Learn About
Alternating Series Test

$A)\quad \sum_{n=1}^{\infty} \frac{1}{n^2}$

Converges $p = 2 > 1$

$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

$S_{100} = 1.6349$

$S_{1000} = 1.6439$

$S_{10000} = 1.6448$

$B)\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

Converges absolutely because the series alternates and the terms decrease in absolute value to 0

$-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16}$

$S_{100} = -.8224$

$S_{1000} = -.8224$

$S_{10000} = -.8224$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

$C)\quad \sum_{n=1}^{\infty} \frac{1}{n} =$

Harmonic Series
Diverges

$S_{100} = 5.187$

$S_{1000} = 7.4854$

$S_{10000} = 8.17836$

$-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots$

$D)\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Alternating Harmonic Series

Conditionally Converges

→ alternating series

$S_{100} = -.6881$

$S_{1000} = -.6926$

$S_{10000} = -.6928$

Conditional Convergence because the series is alternating and the terms decrease in absolute value to 0

	$E)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}} =$ Absolute Convergence? $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ diverges compare to $\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic $\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \frac{n}{\sqrt{n^2+1}} = 1$ (1) $\lim_{n\to\infty} \left\lvert \frac{(-1)^n}{\sqrt{n^2+1}} \right\rvert = 0$ Absolute value of the terms approach 0 (2) $\left\lvert \frac{(-1)^{n+1}}{\sqrt{(n+1)^2+1}} \right\rvert < \left\lvert \frac{(-1)^n}{\sqrt{n^2+1}} \right\rvert$ next term smaller than the previous
Conditional Convergence	$F)\ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}} =$ (1) $\lim_{n\to\infty} \left\lvert \frac{(-1)^{n-1}}{n^{1/3}} \right\rvert = 0$ (2) $\left\lvert \frac{(-1)^{n+1-1}}{(n+1)^{1/3}} \right\rvert < \left\lvert \frac{(-1)^{n-1}}{n^{1/3}} \right\rvert$ Absolute? $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges $p = \frac{1}{3} < 1$
Conditional convergence NO Diverges	$G)\ \sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1} =$ (1) $\lim_{n\to\infty} \left\lvert \frac{(-1)^n n^4}{n^3+1} \right\rvert \neq 0$

Conditional Convergence	$H)\ \sum_{n=1}^{\infty}\frac{\cos(n\pi)}{2^n}=\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n}$ Absolute Convergence $\sum_{n=1}^{\infty}\frac{1}{2^n}$ $r=\frac{1}{2}\quad \lvert r\rvert<1$ Converges ① $\lim_{n\to\infty}\left\lvert\frac{(-1)^n}{2^n}\right\rvert=0$ ② $\left\lvert\frac{(-1)^{n+1}}{2^{n+1}}\right\rvert<\left\lvert\frac{(-1)^n}{2^n}\right\rvert$ $I)\ \sum_{n=1}^{\infty}\frac{(-1)^n n}{\sqrt{n^2+1}}=$ J) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4+2}=$ $K)\ \sum_{n=1}^{\infty}\frac{(-1)^n}{(1.1)^n}$

	8) $\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n} =$ 11) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{3/4}} =$ 12) $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n!} =$ 15) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!} =$