Next Term $\frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^3}{3!}$

$\frac{\frac{\sqrt{2}}{2}\left(\frac{7\pi}{30}-\frac{\pi}{4}\right)^3}{3!}$

4) Write the 2nd order Taylor Polynomial for $f(x) = \cos x$ at $x = \frac{\pi}{4}$.
Then use Taylors Inequality to determine the error bound at $x = 42°$

$\frac{42\pi}{180} = \frac{7\pi}{30}$

$\frac{3\pi}{180}$

3° degrees from center

$f(x) = \cos x$ $\quad f(\pi/4) = \sqrt{2}/2$

$f'(x) = -\sin x$ $\quad f'(\pi/4) = -\sqrt{2}/2$

$f''(x) = -\cos x$ $\quad f''(\pi/4) = -\sqrt{2}/2$

$f'''(x) = \sin x$

$P_2\left(x-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^2}{2!}$

error bound $\leq \frac{\frac{\sqrt{2}}{2}\left(\frac{\pi}{60}\right)^3}{3!}$

Build next term at $x = \frac{7\pi}{30}$

$x = \frac{\pi}{4}$

$\frac{f^3(\pi/4)x^3}{3!} = \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^3}{3!}$

$\frac{f^3\left(\frac{7\pi}{30}\right)x^3}{3!} = \frac{.669\left(x-\pi/4\right)^3}{3!}$

the actual difference is less than .00001697

5) Write the 1st degree Taylor Polynomial for $f(x) = \arcsin x$ at $x = 0$.
Then use Taylors Inequality to determine the error bound at $x = .2$

9) Given that $P_1(x) = x$ represents the first order polynomial for sinx centered at x = 0. Use the Lagrange Error Bound to find the error when $|x| \le .05$

14) Given that $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$ represents the third order Taylor polynomial for ln(x) centered at x = 1. Use the Lagrange Error Bound to find the error when $|x-1| \le .1$

$f(x) = \ln x$

$f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = -x^{-2}$

$f^3(x) = 2x^{-3}$

$f^4(x) = -6x^{-4} = -\frac{6}{x^4}$

.1 away from center

Build next term at

x = .9: $\frac{f^4(.9)(x-1)^4}{4!}$ $\left|\frac{-9.144(x-1)^4}{4!}\right|$

x = 1: $\frac{f^4(1)(x-1)^4}{4!}$ $\left|\frac{-6(x-1)^4}{4!}\right|$

x = 1.1: $\frac{f^4(1.1)(x-1)^4}{4!}$ $\left|\frac{-4.098(x-1)^4}{4!}\right|$

error Bound

$|f(x) - P(x)| \le R$

$R = \frac{\frac{-6}{(.9)^4}(.1)^4}{4!}$

error bound $\le \left|\frac{\frac{-6}{(.9)^4}(.9-1)^4}{4!}\right|$

Summary of Error Bound

For an Alternating Series – Use the next term

For a series that is Not Alternating

1. Write down the formula for the next derivative.

2. Find the value of the next derivative at the ends of the interval and the center.

3. Whichever value is bigger is the value you use to build your error bound term