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Let $f(x)=\dfrac{1}{x-2}$ at x = 3.

$f(x)=\dfrac{1}{x-2} \rightarrow P_3(x-3) = 1-(x-3)+\dfrac{2(x-3)^2}{2!}-\dfrac{6(x-3)^3}{3!}$

a. Write the first 4 terms and the general term of the Taylor Series generated by f(x) at x = 3.

$f(x)=(x-2)^{-1}$ $\quad f(3)=1$

$f'(x)=-(x-2)^{-2}$ $\quad f'(3)=-1$

$f''(x)=2(x-2)^{-3}$ $\quad f''(3)=2$

$f^3(x)=-6(x-2)^{-4}$ $\quad f'''(3)=-6$

general term

$(-1)^n(x-3)^n$

b. Use the result in part (a) ~~to find~~ the fourth order polynomial and the general term of the series generated by $\ln|x-2|$ at x = 3.

$\int f(x) = (x-3) - \frac{1}{2}(x-3)^2 + \frac{1}{3}(x-3)^3 - \frac{1}{4}(x-3)^4$.

$x=.5 \quad -2.5 - \frac{1}{2}(-2.5)^2 + \frac{1}{3}(-2.5)^3 - \frac{1}{4}(2.5)^4$

$\ln|x-2|$

error bound

c. Use the series in part (b) to compute a number that differs from ln(1.5) by less than 0.05. Justify your answer.

next term $= \frac{1}{5}(x-3)^5$ $\qquad$ error bound $\le \frac{1}{5}(x-3)^5$

not alternating $\cancel{x=.5}$ $\qquad$ $x=3.5$ $\qquad$ $\le \boxed{\frac{1}{5}(3.5-3)^5}$

83. The Taylor Series for lnx, centered at x = 1, is $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $.3 \le x \le 1.7$ is

(A) .030 (B) .039 (C) .145 (D) .153 (E) .529

2011 BC6

Let $f(x)=\sin\left(x^2\right)+\cos x$.

a. Write the first four nonzero terms of the Taylor series for sinx about x = 0, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about x = 0.

b. Write the first four nonzero terms of the Taylor series for cosx about x = 0. Use this series and the series for $\sin(x^2)$, found in part a, to write the first four nonzero terms of the Taylor series for f(x) about x = 0.

$f(x) = \sin(x^2) + \cos x$

$f(x) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right)$

$= 1 + \left(x^2 - \frac{x^2}{2!}\right) + \frac{x^4}{4!} + \left(\frac{-x^6}{3!} - \frac{x^6}{6!}\right)$

c. Find the value of $f^{(6)}(0)$.

$\frac{-120x^6}{6!} - \frac{x^6}{6!}$

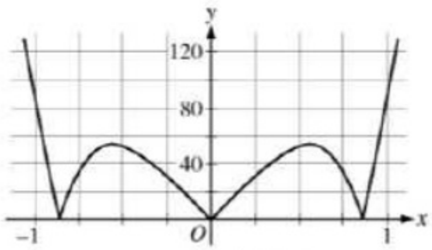
$\frac{-121x^6}{6!}$

cuefficient

6th derivative $\rightarrow$ $\frac{f^6(0)\,x^6}{6!} = \frac{-121x^6}{6!}$

$f^6(0) = -121$

d. Let $f(x)=\sin\left(x^2\right)+\cos x$. The graph of $y=\left|f^{(5)}(x)\right|$ is shown.



Graph of $y=\left|f^{(5)}(x)\right|$

Let $P_4(x)$ be the fourth degree Taylor polynomial for f about x = 0. Using information from the graph of $y=\left|f^{(5)}(x)\right|$, shown above, show that $\left|P_4\left(\frac{1}{4}\right)-f\left(\frac{1}{4}\right)\right|<\frac{1}{3000}$.

2004 BC6

Let f be the function given by $f(x)=\sin\left(5x+\frac{\pi}{4}\right)$, and let P(x) be the third-degree Taylor polynomial for f about x = 0.

a) Find P(x).

b) Find the coefficient of x^{22} in the Taylor series about x = 0.

c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right)-P\left(\frac{1}{10}\right)\right|<\frac{1}{100}$.

d) Let G be the function given $G(x)=\int_0^x f(t)dt$. Write the third-degree Taylor polynomial for G about x = 0.

$f(x)=\sum_{n=0}^{\infty} c_n x^n$	INTERVAL OF CONVERGENCE	RADIUS OF CONVERGENCE
$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\cdots$	$(-1,1)$	1
$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$	$(-\infty,\infty)$	∞
$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$	$(-\infty,\infty)$	∞
$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	$(-\infty,\infty)$	∞
$\tan^{-1} x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$	$(-1,1]$	1
$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$	$(-1,1]$	1