2015 BC6

1. ~~The Maclaurin series for a function f is given by~~

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n} x^n \cdots}$$ and converges to f(x) for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

Sum

$n = 4$

$$\frac{(-3)^{4-1} x^4}{4}$$

$$-\frac{27x^4}{4}$$

a) Use the Ratio Test to find R

$$\lim_{n\to\infty} \left| \frac{(-3)^n x^{n+1}}{n+1} \cdot \frac{n}{(-3)^{n-1} x^n} \right| = \left| \frac{x}{3^{-1}} \right| = |3x|$$

$$-1 < 3x < 1$$

$$-\frac{1}{3} < x < \frac{1}{3}$$

$$R = \frac{1}{3}$$

b) Write the first four non-zero terms of the Maclaurin series for f', the derivative of f. Express f' as a rational function for $|x| < R$.

$$f'(x) = 1 - 3x + 9x^2 - 27x^3$$

$$\text{Sum} = \frac{1}{1 - (-3x)}$$

c) Write the first four nonzero terms of the Maclaurin series for e^x. Use the Maclaurin series for e^x to write the third-degree polynomial for g(x) = $e^x f(x)$ about x = 0.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$g(x) = e^x \cdot f(x)$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)\left(x - \frac{3}{2}x^2 + 3x^3 \dots\right)$$

$$= x - \frac{3}{2}x^2 + 3x^3$$

$$+ 1x^2 - \frac{3}{2}x^3$$

$$+ \frac{1x^3}{2}$$

$$\boxed{x - \frac{1}{2}x^2 + 2x^3}$$

actual difference between lnx and the polynomial	p. 527 #60 Let $f(x) = \frac{1}{x-2}$ at x = 3. a. Write the first 4 terms and the general term of the Taylor Series generated by f(x) at x = 3. b. Use the result in part (a) to find the fourth order polynomial and the general term of the series generated by $\ln\lvert x-2\rvert$ at x = 3. c. Use the series in part (b) to compute a number that differs from ln(1.5) by less than 0.05. Justify your answer. 83. The Taylor Series for lnx, centered at x = 1, is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$. Let f be the function given by ~~the sum~~ of the first three nonzero terms of this series. The maximum value of $\lvert \ln x - f(x) \rvert$ for $.3 \le x \le 1.7$ is (A) .030 (B) .039 (C) .145 (D) .153 (E) .529 $P_3(x-1) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

2011 BC6

Let $f(x) = \sin(x^2) + \cos x$.

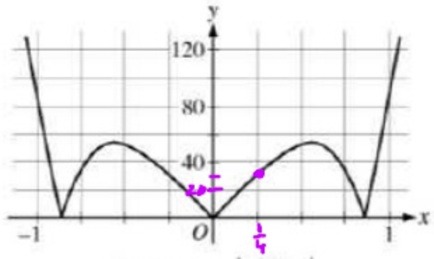
a. Write the first four nonzero terms of the Taylor series for sinx about x = 0, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about x = 0.

b. Write the first four nonzero terms of the Taylor series for cosx about x = 0. Use this series and the series for $\sin(x^2)$, found in part a, to write the first four nonzero terms of the Taylor series for f(x) about x = 0.

c. Find the value of $f^{(6)}(0)$.

d. Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = \left|f^{(5)}(x)\right|$ is shown.

$\left| 5^{th} \text{ derivative} \right|$



Graph of $y = \left|f^{(5)}(x)\right|$

$$\boxed{\frac{40\left(\frac{1}{4}\right)^5}{5!}}$$

Let $P_4(x)$ be the fourth degree Taylor polynomial for f about x = 0. Using information from the graph of $y = \left|f^{(5)}(x)\right|$, shown above, show that

$$\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}.$$

error bound

Build next term at $x = \frac{1}{4}$

$\cancel{x = 0}$

$$\cancel{\frac{f^5(0)x^5}{5!}}$$

$$\frac{f^5\left(\frac{1}{4}\right)x^5}{5!}$$

2004 BC6

Let f be the function given by $f(x)=\sin\left(5x+\frac{\pi}{4}\right)$, and let P(x) be the third-degree Taylor polynomial for f about x = 0.

a) Find P(x).

b) Find the coefficient of x^{22} in the Taylor series about x = 0.

c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right)-P\left(\frac{1}{10}\right)\right|<\frac{1}{100}$.

d) Let G be the function given $G(x)=\int_0^x f(t)dt$. Write the third-degree Taylor polynomial for G about x = 0.

$f(x)=\sum_{n=0}^{\infty} c_n x^n$	INTERVAL OF CONVERGENCE	RADIUS OF CONVERGENCE
$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\cdots$	$(-1,1)$	1
$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$	$(-\infty,\infty)$	∞
$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$	$(-\infty,\infty)$	∞
$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	$(-\infty,\infty)$	∞
$\tan^{-1} x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$	$(-1,1]$	1
$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$	$(-1,1]$	1