What you'll Learn About
How to write terms given a power series
How to take the derivative and anti-derivative of a power series
Identifying important types of power series

1. Given the series $\sum_{n=0}^{\infty} x^n$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty} x^n =$$

b. Determine the Interval of Convergence of the series.

c. Determine the function (sum) of the series (f(x) =)

d. Enter the first 6 terms into y_1 and the function (sum) into y_2 of your calculator.

e. Set the x-values of your window to match your interval of convergence and the y-values from [0,10]. What do you notice about the 2 graphs?

f. What would happen to the graphs if the first 10 terms of the series are entered into y_1.

i. Take the derivative of $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots x^n = \frac{1}{1-x}$

j. Take the anti- derivative of $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots x^n = \frac{1}{1-x}$

	2. Given the series $\sum_{n=0}^{\infty}(-1)^n (x)^n$ answer the following questions. a. List the first 6 terms of the series and the general term $\sum_{n=0}^{\infty}(-1)^n (x)^n =$ b. What do you notice about the terms of the sequence compared to the terms in number 1. c. Determine the Interval of Convergence of the series. d. Determine the function (sum) of the series (f(x) =) e. Take the derivative of $\sum_{n=0}^{\infty}(-1)^n (x^n) = 1 - x + x^2 - x^3 + \cdots (-1)^n (x^n) = \frac{1}{1+x}$ f. Take the anti- derivative of $\sum_{n=0}^{\infty}(-1)^n (x^n) = 1 - x + x^2 - x^3 + \cdots (-1)^n (x^n) = \frac{1}{1+x}$

	3. Given the series $\sum_{n=0}^{\infty}(-1)^n(x)^{2n}$ answer the following questions. a. List the first 6 terms of the series and the general term $\sum_{n=0}^{\infty}(-1)^n(x)^{2n} =$ b. What do you notice about the terms of the sequence compared to the terms in number 2a. c. Determine the Interval of Convergence of the series. d. Determine the function (sum) of the series (f(x) =) e. Take the derivative of $\sum_{n=0}^{\infty}(-1)^n(x)^{2n} = 1-x^2+x^4-x^6+\cdots(-1)^n(x)^{2n}\cdots = \frac{1}{1+x^2}$ k. Take the anti- derivative of $\sum_{n=0}^{\infty}(-1)^n(x)^{2n} = 1-x^2+x^4-x^6+\cdots(-1)^n(x)^{2n}\cdots = \frac{1}{1+x^2}$

4. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} =$$

b. Use the Ratio Test to determine the Interval of Convergence of the series.

c. Enter the first 6 terms into y_1 of your calculator. Use X[-π,π]$_1$ and Y[-1,1] as your window.

d. What function does it look like the series represents? That function is the sum of this series.

e. What would happen to the graphs if the first 10 terms of the series are entered into y_1.

f. Take the derivative of $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \cos x$

i. Take the anti-derivative of $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \cos x$

5. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} =$$

b. Use the Ratio Test to determine the Interval of Convergence of the series.

c. Enter the first 6 terms into y_1 of your calculator. Use $X[-\pi,\pi]_1$ and $Y[-1,1]$ as your window.

d. What function does it look like the series represents? That function is the sum of this series.

e. Take the derivative of

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots - (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sin x$$

f. Compare the derivative of the series in part f to the series you found in problem 4a.

g. Take the anti-derivative of

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots - (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sin x$$

	6. Given the series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ answer the following questions. a. List the first 6 terms of the series and the general term $\sum_{n=0}^{\infty}\frac{x^n}{n!}=$ b. Use the Ratio Test to determine the Interval of Convergence of the series. c. Enter the first 6 terms into y_1 of your calculator. Use $X[-3,3]_1$ and Y[0,10] as your window. d. What function does it look like the series represents? That function is the sum of this series. f. Take the derivative of $\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots=e^x$ g. Take the anti-derivative of $\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots=e^x$

7. Given the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}=$$

b. Use the Ratio Test to determine the Interval of Convergence of the series. Don't forget to test your endpoints.

c. Enter the first 6 terms into y_1 of your calculator. Use your interval of convergence for your x window and Y[-2,2] as your window.

d. What function does it look like the series represents? That function is the sum of this series.

f. Take the derivative of $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n-1}\frac{x^n}{n}+\cdots=\ln(1+x)$

g. Compare your derivative in part f to the series you wrote in problem 2a.

h. Take the anti-derivative of

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n-1}\frac{x^n}{n}+\cdots=\ln(1+x)$$

8. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} =$$

b. Use the Ratio Test to determine the Interval of Convergence of the series. Don't forget to test your endpoints.

c. Enter the first 6 terms into y_1 of your calculator. Use your interval of convergence for your x window and $Y\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ as your window.

d. What function does it look like the series represents? That function is the sum of this series.

f. Take the derivative of

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \arctan(x)$$

g. Compare your derivative in part f to the series you wrote in problem 3a.

h. Take the anti-derivative of $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots =$

What you'll Learn About
Taking derivatives and anti-derivatives of a power series

1. Find the first 4 terms of the series
2. Write the rule for the series
3. Find the interval of convergence
4. Take the derivative of the function and series
5. Take the anti-derivative of the function and the series

Geometric Series

1) $f(x)=\frac{1}{1+x^3}$

2) $f(x)=\frac{x^3}{1-x^2}$

1. Find the first 4 terms of the series 2. Write the rule for the series 3. Find the interval of convergence 4. Take the derivative of the function and series 5. Take the anti-derivative of the function and the series	3) $f(x)=\sin(x^2)$ 4) $f(x)=x^2\cos(x^3)$

1. Find the first 4 terms of the series 2. Write the rule for the series 3. Find the interval of convergence 4. Take the derivative of the function and series 5. Take the anti-derivative of the function and the series	5) $f(x) = x^2e^{x^3}$ 6) $f(x) = xe^{x^4}$

1. Find the first 4 terms of the series 2. Write the rule for the series 3. Find the interval of convergence 4. Take the derivative of the function and series 5. Take the anti-derivative of the function and the series	7) $f(x)=x^2\tan^{-1}\left(x^5\right)$ 8) $f(x)=\ln\left(1-x^4\right)$

	Construct the first 3 nonzero terms and the general term of the Maclaurin Series generated by the function and give the interval of convergence. A) $f(x) = e^{5x}$ B) $f(x) = \ln(1+2x)$ C) $f(x) = \dfrac{1}{x+5}$

	Determine which value the series converges to. (Determine the value of the series/Determine the sum of the series) $A)\ \sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^{n}$ $B)\ \sum_{n=0}^{\infty}\frac{5^{n}}{n!}$ $C)\ \sum_{n=0}^{\infty}\frac{(-1)^{n}\left(\frac{\pi}{4}\right)^{2n+1}}{(2n+1)!}$ $D)\ \sum_{n=0}^{\infty}\frac{(-1)^{n}\left(\frac{1}{2}\right)^{2n+1}}{2n+1}$

What you'll Learn About
How to build a polynomial using derivatives

$P(0) = 7$
$P'(0) = 3$
$P''(0) = 9$
$P'''(0) = 15$
$P^4(0) = 6$
$P^5(0) = 4$
$P^6(0) = 12$

Given the values of the following, construct the 6th degree Taylor Polynomial centered at x = 0

$P(x) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$

What would be the next 2 terms if $P^7(0) = 22$ and $P^8(0) = 50$?

	Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = 0 1. $P(0)=2\;\; P'(0)=5\;\; P''(0)=8\;\; P'''(0)=11\;\; P^{4}(0)=14$ 2. $P(0)=5\;\; P'(0)=-2\;\; P''(0)=7\;\; P'''(0)=-4\;\; P^{4}(0)=10$ Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = 2 3. $P(2)=2\;\; P'(2)=5\;\; P''(2)=8\;\; P'''(2)=11\;\; P^{4}(2)=14$ Given the values of the following, construct the 4th degree Taylor Polynomial centered at x = -2 4. $P(-2)=5\;\; P'(-2)=-2\;\; P''(-2)=7\;\; P'''(-2)=-4\;\; P^{4}(-2)=10$

	Create the Maclaurin Series for $f(x) = e^x$ by using the Taylor Polynomial process Create the Maclaurin Series for $f(x) = \sin x$ by using the Taylor Polynomial process

	Write the first four terms for $f(x) = \sin(x)$ Find each of the following 1. $f^{(5)}(0) =$ 2. $f^{(13)}(0) =$ 3. $f^{(23)}(0) =$ Write the first four terms for $f(x) = \sin(x^4)$ Find each of the following 1. $f^{(4)}(0) =$ 2. $f^{(12)}(0) =$ 3. $f^{(20)}(0) =$

	$\text{If } g(x) = \cdots \frac{x^{12}}{12} \cdots \text{ find } f^{12}(0)$ $\text{If } g(x) = \cdots \frac{(x-3)^{20}}{10} \cdots \text{ find } f^{20}(3)$

	Find the 3rd order Taylor Polynomial centered at x = 2 18) $f(x) = \frac{1}{x}$ 19) $f(x) = \sin x$ at $x = \frac{\pi}{4}$

Sometimes if your center is not at zero, you do not need to build the polynomial. You can use the MacLaurin series and Substitution.

Construct the polynomial for $f(x) = e^{x-1}$ centered at x =1 using derivatives.

Construct the polynomial for $f(x) = e^{x-1}$ centered at x = 1 using a MacLaurin series.

What you'll Learn About
How to find the error for a series that alternates

1. Give the first term of the series for f(x) = arctan(x) centered at x = 0
2. Find the approximation for P(.1)
3. Find the f(.1)
4. How accurate is the approximation.
5. What is the value of the next term of the polynomial at x =.1

1. Give the first 2 terms of the series for f(x) = arctan(x) centered at x = 0
2. Find the approximation for P(.1)
3. Find the f(.1)
4. How accurate is the approximation.
5. What is the value of the next term of the polynomial at x =.1

1. Give the first 3 terms of the series for f(x) = arctan(x) centered at x = 0
2. Find the approximation for P(.1)
3. Find the f(.1)
4. How accurate is the approximation.
5. What is the value of the next term of the polynomial at x =.1

	1. Give the first 4 terms of the series for f(x) = arctan(x) centered at x = 0 *2. Use the alternate estimation theorem to determine the error bound* $\lvert f(x) - P(x) \rvert \le R$ 1. Give the first 4 terms of the series for f(x) = sin(x) centered at x = $\frac{\pi}{2}$ 2. Use the alternate estimation theorem to determine the error bound at x =1.6 $\lvert f(x) - P(x) \rvert \le R$

	1. $f(x) = \frac{1}{x}$ centered at x = 2 a. Given the function, find the fourth order polynomial b. Use the alternate estimation theorem to find a formula for the error bound $\lvert f(x) - P(x) \rvert \le R$

CALCULUS: Graphical,Numerical,Algebraic by Finney,Demana,Watts and Kennedy
Chapter 9: Error and Series 9.3:

What you'll Learn About
How to find the error of a series that does not alternate

Lagrange Error Bound/Taylors Inequality/Remainder Estimation Theorem

1. Give the first term of the series for $f(x) = e^x$ centered at $x = 0$

2. Find the approximation for P(.1)

3. Find f(.1)

4. How accurate is the approximation.

5. What is the value of the next term of the polynomial at x =.1

1. Give the first two terms of the series for $f(x) = e^x$ centered at $x = 0$

2. Find the approximation for P(.1)

3. Find f(.1)

4. How accurate is the approximation.

5. What is the value of the next term of the polynomial at x =.1

1. Give the first three terms of the series for $f(x) = e^x$ centered at $x = 0$

2. Find the approximation for P(.1)

3. Find f(.1)

4. How accurate is the approximation.

5. What is the value of the next term of the polynomial at x =.1

$\lvert f(x)-P(x)\rvert \le R$ *Where* *R* = $\left\lvert \dfrac{\begin{pmatrix} Max \text{ of the} \\ \text{next derivative} \\ \text{on the given} \\ \text{interval} \end{pmatrix}(x-c)^{n+1}}{(n+1)!} \right\rvert$	1. *Give the first 4 terms of the series for f(x)=e^x centered at x = 0* 2. *Use Taylors Inequality to determine the error bound* $\lvert f(x)-P(x)\rvert \le R$
Where **x-c** is the distance from the center Where **n** is the order We must build the next term a little bit bigger to have a good boundary for the error. Remember, whenever you see this, $\lvert f(x)-P(x)\rvert \le R$, *you are finding error bound* whenever you see this, $\lvert f(x)-P(x)\rvert$, *you are finding the actual error between the function and the approximation from the polynomial*	1. *Find the 3rd order polynomial of the series for* $f(x)=\dfrac{1}{(1-x)^2}$ *centered at x = 0* 2. *Find the Lagrange error bound* $\lvert f(x)-P(x)\rvert \le R$ *for the series between* $0 \le x \le .2$

4) Write the 2nd order Taylor Polynomial for $f(x) = \cos x$ at $x = \frac{\pi}{4}$.

Then use Taylors Inequality to determine the error bound at $x = 42°$

5) Write the 1st degree Taylor Polynomial for $f(x) = \arcsin x$ at $x = 0$.
Then use Taylors Inequality to determine the error bound at $x = .2$

	9) Given that $P_1(x) = x$ represents the first order polynomial for sinx centered at x = 0. Use the Lagrange Error Bound to find the error when $\lvert x \rvert \le .05$ 14) Given that $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$ represents the third order Taylor polynomial for ln(x) centered at x = 1. Use the Lagrange Error Bound to find the error when $\lvert x-1 \rvert \le .1$

Summary of Error Bound

For an Alternating Series – Use the next term

For a series that is Not Alternating

1. Write down the formula for the next derivative.

2. Find the value of the next derivative at the ends of the interval and the center.

3. Whichever value is bigger is the value you use to build your error bound term

2015 BC6

1. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n} x^n \cdots$ and converges to f(x) for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

   a) Use the Ratio Test to find R

   b) Write the first four non-zero terms of the Maclaurin series for f', the derivative of f. Express f' as a rational function for $|x| < R$.

   c) Write the first four nonzero terms of the Maclaurin series for e^x. Use the Maclaurin series for e^x to write the third-degree polynomial for g(x) = $e^x f(x)$ about x = 0.

Let f b a function that has derivatives of all orders for all real numbers. Assume f(0) = 4, $f'(0) = 5$, $f''(0) = -8$, and $f'''(0) = 6$.

a. Write the third order Taylor Polynomial for f at x = 0 and use it to approximate f(.2).

b. Write the second order Taylor polynomial for f', at x = 0

c. Write the fourth order Taylor polynomial for $\int_0^x f(t)dt$, at x = 0.

d. Determine if the linearization of f is an underestimate or overestimate near 0.

p. 527 57

a. Write the first three nonzero terms and the general term of the Taylor Series generated by $f(x) = 5\sin\left(\frac{x}{2}\right)$ at x = 0.

c. What is the minimum number of terms of the series in part a needed to approximate f(x) on the interval (-2, 2) with an error not exceeding .1 in magnitude. Explain your answer.

p. 492 #24

The Maclaurin Series for f(x) is $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots \frac{x^n}{(n+1)!}$.

a. Find $f'(0)$ and $f^{10}(0)$.

b. Let g(x) = xf(x). Write the Maclaurin Series for g(x), showing the first three non-zero terms and the general term.

c. Write g(x) in terms of a familiar function without using series.

p. 500 #13

Find a formula for the truncation error if we use $P_6(x)$ to approximate $\frac{1}{1+2x}$ on (-.5, .5).

p. 500 20

a. If cos(x) is replaced by $1-\frac{x^2}{2}$ and $|x|<.5$, what estimate can be made of the error?

b. Does $1-\frac{x^2}{2}$ tend to be to large or to small.

p. 500 #22

The approximation $\sqrt{1+x}\approx 1+\frac{x}{2}$ is used when x is small. Estimate the error when $|x|<.1$

p. 527 #60

Let $f(x)=\dfrac{1}{x-2}$ at x = 3.

a. Write the first 4 terms and the general term of the Taylor Series generated by f(x) at x = 3.

b. Use the result in part (a) to find the fourth order polynomial and the general term of the series generated by $\ln|x-2|$ at x = 3.

c. Use the series in part (b) to compute a number that differs from ln(1.5) by less than 0.05. Justify your answer.

83. The Taylor Series for lnx, centered at x = 1, is $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $.3 \le x \le 1.7$ is

(A) .030 (B) .039 (C) .145 (D) .153 (E) .529

2011 BC6

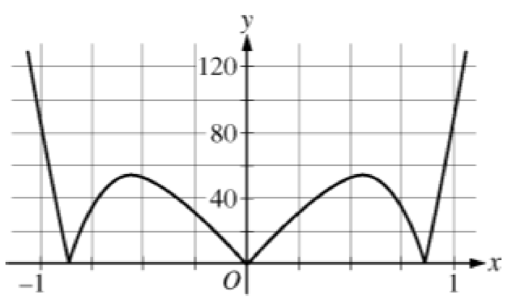
Let $f(x) = \sin(x^2) + \cos x$.

a. Write the first four nonzero terms of the Taylor series for sinx about x = 0, and write the first four nonzero terms of the Taylor series for $\sin(x^2)$ about x = 0.

b. Write the first four nonzero terms of the Taylor series for cosx about x = 0. Use this series and the series for $\sin(x^2)$, found in part a, to write the first four nonzero terms of the Taylor series for f(x) about x = 0.

c. Find the value of $f^{(6)}(0)$.

d. Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = \left|f^{(5)}(x)\right|$ is shown.



Graph of $y = \left|f^{(5)}(x)\right|$

Let $P_4(x)$ be the fourth degree Taylor polynomial for f about x = 0. Using information from the graph of $y = \left|f^{(5)}(x)\right|$, shown above, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

	2004 BC6 Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let P(x) be the third-degree Taylor polynomial for f about x = 0. a) Find P(x). b) Find the coefficient of x^{22} in the Taylor series about x = 0. c) Use the Lagrange error bound to show that $\left\lvert f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right\rvert < \frac{1}{100}$. d) Let G be the function given $G(x) = \int_0^x f(t)dt$. Write the third-degree Taylor polynomial for G about x = 0.

$f(x)=\sum_{n=0}^{\infty} c_n x^n$	INTERVAL OF CONVERGENCE	RADIUS OF CONVERGENCE
$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\cdots$	$(-1,1)$	1
$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$	$(-\infty,\infty)$	∞
$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ $=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$	$(-\infty,\infty)$	∞
$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$ $=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	$(-\infty,\infty)$	∞
$\tan^{-1} x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$ $=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$	$(-1,1]$	1
$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ $=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$	$(-1,1]$	1