What you'll Learn About
How to find the error for a series that alternates

1. Give the first term of the series for f(x) = arctan(x) centered at x = 0

$P_1(x) = x$

2. Find the approximation for P(.1) $= .1$

3. Find f(.1) $= \arctan(.1) = .0996686525$

4. How accurate is the approximation. $.0003313475$ Actual

5. What is the value of the next term of the polynomial at x =.1

next term

next term $= -\frac{x^3}{3} = \frac{-.1^3}{3} = -.000333333$

1. Give the first 2 terms of the series for f(x) = arctan(x) centered at x = 0

$x - \frac{x^3}{3}$

2. Find the approximation for P(.1) $= .09966666$

3. Find the f(.1) $= .0996686525$

4. How accurate is the approximation. $.0000019858$

5. What is the value of the next term of the polynomial at x =.1

next term $= \frac{x^5}{5} \rightarrow \frac{.1^5}{5} = .000002$

$x - \frac{x^3}{3} + \frac{x^5}{5}$

1. Give the first 3 terms of the series for f(x) = arctan(x) centered at x = 0

2. Find the approximation for P(.1) $= .09966867$

3. Find the f(.1) $= .0996686525$

4. How accurate is the approximation. $.000000014175505$

5. What is the value of the next term of the polynomial at x =.1

next term $= \frac{-x^7}{7} \rightarrow \frac{-.1^7}{7} = -.0000000142857143$

1. Give the first 4 terms of the series for f(x) = arctan(x) centered at x = 0

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$$

2. Use the alternate estimation theorem to determine the error bound $|f(x) - P(x)| \le R$

Actual Difference $\le$ Error Bound

next term $\frac{x^9}{9}$

error bound $\frac{.1^9}{9}$

1. Give the first 4 terms of the series for f(x) = sin(x) centered at x = $\frac{\pi}{2}$

$f(x) = \sin x$ $\quad f(\frac{\pi}{2}) = 1$

$f'(x) = \cos x$ $\quad f'(\frac{\pi}{2}) = 0$

$f''(x) = -\sin x$ $\quad f''(\frac{\pi}{2}) = -1$

$f'''(x) = -\cos x$ $\quad f'''(\frac{\pi}{2}) = 0$

$f^4(x) = \sin x$ $\quad f^4(\pi/2) = 1$

$f^5(x) = \cos x$ $\quad f^5(\pi/2) = 0$

$f^6(x) = -\sin x$ $\quad f^6(\pi/2) = -1$

2. Use the alternate estimation theorem to determine the error bound at x =1.6 $|f(x) - P(x)| \le R$

$$P_6(x-\tfrac{\pi}{2}) = 1 - \frac{1(x-2)^2}{2!} + \frac{1(x-\frac{\pi}{2})^4}{4!} - \frac{1(x-\frac{\pi}{2})^6}{6!}$$

$$\text{next term} = \frac{(x-\frac{\pi}{2})^8}{8!}$$

$$\text{error bound} = \frac{(1.6-\frac{\pi}{2})^8}{8!}$$

1. $f(x) = \frac{1}{x}$ centered at x = 2

a. Given the function, find the fourth order polynomial

$f(x) = \frac{1}{x} = x^{-1}$ $\quad f(2) = 1/2$ $\quad f^4(x) = 24x^{-5} = \frac{24}{x^5}$

$f'(x) = -x^{-2} = -1/x^2$ $\quad f'(2) = -1/4$ $\quad f^4(2) = \frac{24}{32}$

$f''(x) = 2x^{-3} = 2/x^3$ $\quad f''(2) = 2/8$ $\quad f^5(x) = -120x^{-6}$

$f^3(x) = -6x^{-4} = -6/x^4$ $\quad f'''(2) = -6/16$ $\quad = \frac{-120}{x^6}$

b. Use the alternate estimation theorem to find a formula for the error bound $|f(x) - P(x)| \le R$

$$P_4(x-2) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{\frac{2}{8}(x-2)^2}{2!} - \frac{\frac{6}{16}(x-2)^3}{3!} + \frac{\frac{24}{32}(x-2)^4}{4!}$$

$$\text{Next Term} = \frac{-f^5(2)(x-2)^5}{5!} = \boxed{\left| \frac{-\frac{120}{2^6}(x-2)^5}{5!} \right|}$$

	Summary of Error Bound For an Alternating Series – Use the next term For a series that is Not Alternating 1. Write down the formula for the next derivative. 2. Find the value of the next derivative at the ends of the interval and the center. 3. Whichever value is bigger is the value you use to build your error bound term