

2015 AP Calculus BC Free Response

6. The Maclaurin Series for a function f is given by

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \dots + \frac{(-3)^{n-1}}{n} x^n + \dots$$

and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

a) Use the Ratio Test to find R .

$$\lim_{n \rightarrow \infty} \left| \frac{(-3)^n x^{n+1}}{(n+1)} \cdot \frac{n}{(-3)^{n-1} x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x \cdot n}{3^{-1}(n+1)} \right| = |3x| < 1$$

Multiple of Harmonic

Interval of Convergence

$$x = -\frac{1}{3} \quad \sum_{n=1}^{\infty} \frac{(-3)^{n-1} \left(\frac{1}{3}\right)^n}{n} = \sum_{n=1}^{\infty} \frac{(-3)^n (-3)^{-1} \left(\frac{1}{3}\right)^n}{n} = \sum_{n=1}^{\infty} \frac{-1}{3n}$$

$$x = \frac{1}{3} \quad \sum_{n=1}^{\infty} \frac{(-3)^{n-1} \left(\frac{1}{3}\right)^n}{n} = \sum_{n=1}^{\infty} \frac{(-3)^n (-3)^{-1} \left(\frac{1}{3}\right)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n \left(\frac{-1}{3n}\right)}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n}$$

Multiple of alternating harmonic

$$R = \frac{1}{3}$$

$$-1 < 3x < 1$$

$$-\frac{1}{3} < x < \frac{1}{3}$$

$$-\frac{1}{3} < x < \frac{1}{3}$$

$$x = \frac{1}{3}$$

conditional convergence

20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converges?

- (A) $-3 \leq x \leq 3$ (B) $-3 < x < 3$ (C) $-1 < x \leq 5$ (D) $-1 \leq x \leq 5$ (E) $-1 \leq x < 5$

