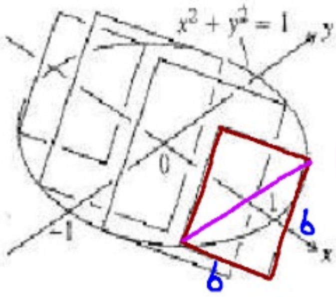
Find the volume of the solid which lies between planes perpendicular to the x-axis at x = -1 and x = 1 between the semi-circles $y=-\sqrt{1-x^2}$ and $y=\sqrt{1-x^2}$. The cross sections perpendicular to the x-axis are squares with diagonals running from $y=-\sqrt{1-x^2}$ to $y=\sqrt{1-x^2}$.



Bottom: $y=-\sqrt{1-x^2}$, Top: $y=\sqrt{1-x^2}$

diagonal = Top − Bottom

$\sqrt{1-x^2}-(-\sqrt{1-x^2})$

$2\sqrt{1-x^2}$

$b^2+b^2=(2\sqrt{1-x^2})^2$

$\frac{2b^2}{2}=\frac{4(1-x^2)}{2}$

$b^2=2(1-x^2)$ → Area of square

$V=\int_{-1}^{1}(\text{Area of Square})\,dx$

$V=\int_{-1}^{1}2(1-x^2)\,dx$

The solid lies between planes perpendicular to the x-axis at x = 0 and x = 4. the cross sections perpendicular to the x-axis between these planes run from $y=-\sqrt{x}$ and $y=\sqrt{x}$. If the cross-sections are equilateral triangles with one side running from $y=-\sqrt{x}$ and $y=\sqrt{x}$

side of Δ = Top − Bottom

Side = $\sqrt{x}-(-\sqrt{x})$

side = $2\sqrt{x}$

$V=\int_0^4(\text{Area of a }\Delta)\,dx$

$V=\int_0^4\frac{1}{2}(2\sqrt{x})(\sqrt{3x})\,dx=\int_0^4\sqrt{x}\cdot\sqrt{3}\sqrt{x}$

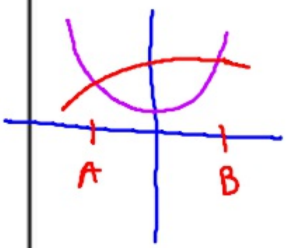
$=\sqrt{3}\int_0^4 x\,dx$

$h^2+(\sqrt{x})^2=(2\sqrt{x})^2$

$h^2+x=4x$

$h^2=3x$

$h=\sqrt{3x}$

	Let R be the region in quadrant I and II enclosed by the graphs of $y = 2 + \sin(x)$, $y = \sec(x)$
Intersection $A = -.725444$ $B = 1.2237831$ 	a) Find the volume of a solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares. $V = \int_A^B (2+\sin x - \sec x)^2 dx = \int_A^B (2+\sin x - \sec x)(2+\sin x - \sec x)$
Base = Top-bottom →	b) Find the volume of a solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are isosceles right triangles. $V = \int_A^B \frac{1}{2}(2+\sin x - \sec x)^2 dx$
Radius = Top-Bottom	c) Find the volume of a solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are circles. $\int_A^B \pi \left(\frac{2+\sin x - \sec x}{2}\right)^2 dx$
radius = Top-Bottom	d) Find the volume of a solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are semi-circles. $\int_A^B \frac{1}{2}\pi \left(\frac{2+\sin x - \sec x}{2}\right)^2 dx$

Direction

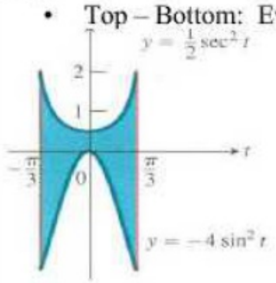
- A particle is stopped when the velocity = 0
- A particle moves left when the velocity is negative
- A particle moves right when the velocity is positive

Displacement/Total Distance

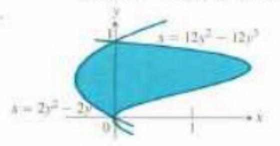
- Displacement is the integral of the velocity
- Total Distance is the integral of the absolute value of the velocity
    - Remember when doing total distance by hand you must find when the particle is moving left and right and split up your integral doing the absolute value of the part that is moving left

Area

- Top – Bottom: Everything in the integral is in terms of x



- Right – Left: Everything in the integral is in terms of y.



Arc Length

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$ if original equation is solved for y

$$L = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$ if original equation is solved for x

$V = \int (\text{Area of the shape})\, dx$

Top – bottom

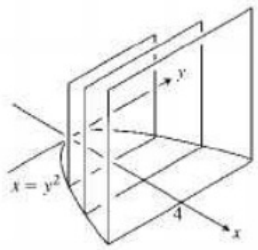
base of a square

base of Δ

diameter of circle

Volume of a cross-section

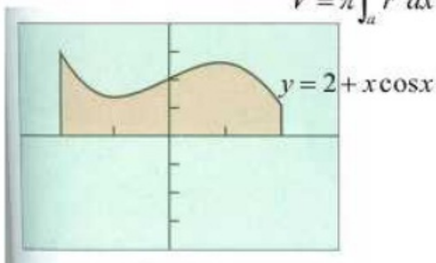
(b) The cross sections are squares with bases in the xy-plane.



$$V = \int_a^b s^2 dx = \int_a^b \left(2\sqrt{x}\right)^2 dx$$

Volume using disks

$$V = \pi \int_a^b r^2 dx \quad \text{or} \quad V = \pi \int_a^b r^2 dy$$



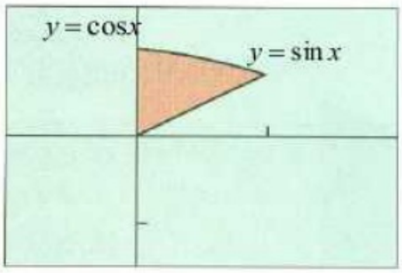
[−3, 3] by [−4, 4]

Rotate about the x-axis

Disks will occur when your shaded region is flat against the line that you are rotating around.

Radius is always the curve

Volume using washers occurs when the shaded region is not flat against the line of rotation



$$V = \pi\int_a^b (OuterRing)^2 - (InnerRing)^2$$

[−π/4, π/2] by [−1.5, 1.5]

Rotate about the x-axis

- If you rotate around the x-axis or a line y = a, then everything is in terms of x (solved for y)
- If you rotate around the y-axis or a line x = a, then everything is in terms of y (solved for x)
- Washers occur when your shaded region does not touch the line you are rotating about.

Summary of rotating about a line y = a (similar to x-axis):
Radius Always Top - Bottom

- If the original curve is below y = a, then the radius is the line minus the curve
- If the original curve is above y = a then the radius is the curve minus the line

Summary of rotating about a line x = a (similar to y-axis):
Radius Always Right - Left

- If the original curve is to the left of x = a, then the radius is the line minus the curve
- If the original curve is to the right of x = a then the radius is the curve minus the line