Page 9

What you'll Learn About

- Finding volume of a solid using square/circular/triangular cross-sections
- Finding volume of a solid rotated about an axis or line

Find the volume of the solid which lies between planes perpendicular to the x-axis at x = -1 and x =1 between the semi-circles $y=-\sqrt{1-x^2}$ and $y=\sqrt{1-x^2}$. The cross sections perpendicular to the x-axis are squares with one side in the disk.

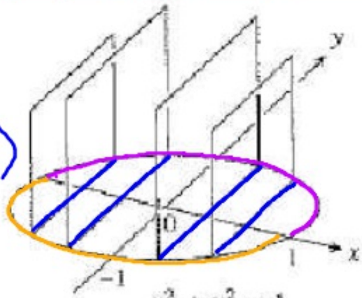
Bottom

Top

Base = Top - Bottom

$$\text{Base} = \sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right)$$

$$= 2\sqrt{1-x^2}$$



$x^2+y^2=1$

$$V = \int_{-1}^{1} b \cdot h \cdot \text{thickness}$$

$$V = \int_{-1}^{1} \underbrace{\left(2\sqrt{1-x^2}\right)\left(2\sqrt{1-x^2}\right)}_{\text{Area}} dx$$

$$V = \int_{-1}^{1} (\text{Area}) \text{ thickness}$$

Find the volume of the solid which lies between planes perpendicular to the y-axis at y = -1 and y =1 between the semi-circles $x=-\sqrt{1-y^2}$ and $x=\sqrt{1-y^2}$. The cross sections perpendicular to the y-axis are isosceles triangles with one leg in the disk.

Left

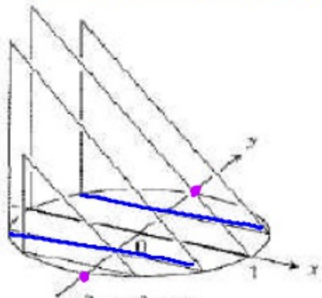
Right

$\frac{1}{2} b h$

base = right - left

$$= \sqrt{1-y^2} - \left(-\sqrt{1-y^2}\right)$$

$$= 2\sqrt{1-y^2}$$



$x^2+y^2=1$

$$V = \int (\text{Area}) \text{ thickness}$$

$$V = \int_{-1}^{1} \frac{1}{2}\left(2\sqrt{1-y^2}\right)\left(2\sqrt{1-y^2}\right) dy$$

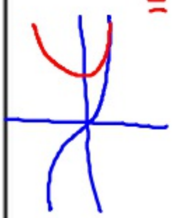
$$V = \int_{-1}^{1} 2(1-y^2) = \int_{-1}^{1} 2-2y^2$$

$$= 2y - \frac{2}{3}y^3 \Big]_{-1}^{1}$$

40a. The solid lies between planes perpendicular to the x-axis at $x=\frac{-\pi}{3}$ and $x=\frac{-\pi}{3}$. The cross sections perpendicular to the x-axis are circular disks with diameters running from the curve **y = tanx** to the curve **y = secx**. Find the volume of the solid.

$A=\pi r^2$

diameter = Top − Bottom



$= \sec x - \tan x$

$$V = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (\text{Area of a circle})\ \text{thickness}$$

$$V = \int_{-\frac{\pi}{3}}^{\pi/3} \pi\left(\frac{\sec x - \tan x}{2}\right)^2 dx$$

40b. The solid lies between planes perpendicular to the x-axis at $x=\frac{-\pi}{3}$ and $x=\frac{-\pi}{3}$. The cross sections perpendicular to the x-axis are squares whose bases run from the curve **y = tanx** to the curve **y = secx**. Find the volume of the solid.

$A = b \cdot h$

base = sec x − tan x

$$\boxed{V = \int_{-\frac{\pi}{3}}^{\pi/3} (\text{Area of Square})\ \text{thickness}}$$

$$V = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (\sec x - \tan x)^2\, dx$$

Direction

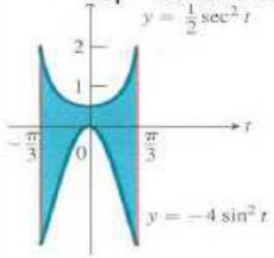
- A particle is stopped when the velocity = 0
- A particle moves left when the velocity is negative
- A particle moves right when the velocity is positive

Displacement/Total Distance

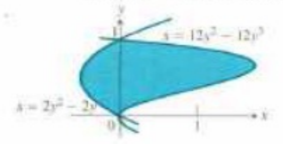
- Displacement is the integral of the velocity
- Total Distance is the integral of the absolute value of the velocity
    - Remember when doing total distance by hand you must find when the particle is moving left and right and split up your integral doing the absolute value of the part that is moving left

Area

- Top – Bottom: Everything in the integral is in terms of x



- Right – Left: Everything in the integral is in terms of y.



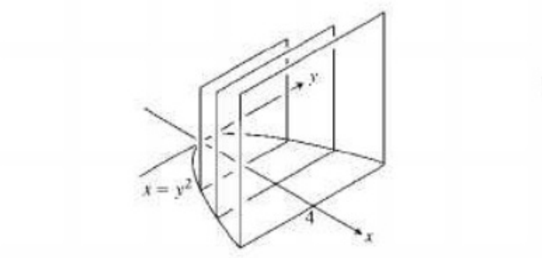
Arc Length

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \text{ if original equation is solved for y}$$

$$L = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \text{ if original equation is solved for x}$$

Volume of a cross-section

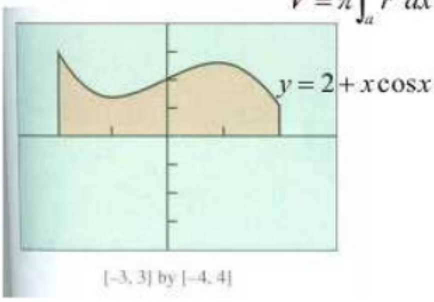
(b) The cross sections are squares with bases in the xy-plane.



$$V=\int_a^b s^2\,dx=\int_a^b\left(2\sqrt{x}\right)^2 dx$$

Volume using disks

$$V=\pi\int_a^b r^2\,dx \quad \text{or} \quad V=\pi\int_a^b r^2\,dy$$



[−3, 3] by [−4, 4]

Rotate about the x-axis

Disks will occur when your shaded region is flat against the line that you are rotating around.

Radius is always the curve