

2010 #1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

rate snow accumulates →

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t \leq 7 \\ 108 & \text{for } 7 \leq t \leq 9 \end{cases}$$

rate removing snow →

$$\rightarrow h(t) = \int g(t)$$

a. How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

$$\text{Total} = \int_0^6 7te^{\cos t} dt = 142.274$$

b. Let $h(t)$ represent the total amount of snow in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \leq t \leq 9$.

$$h(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125(t-6) & 6 \leq t < 7 \\ \cancel{108(t-7)} & 7 \leq t < 9 \end{cases}$$

$$h(7) = 125$$

$$\int 108 dt$$

$$h(t) = 108t + C$$

$$h(t) = 108t - 631$$

$$\begin{aligned} 125 &= 108(7) + C \\ 125 &= 756 + C \\ -756 & \quad -756 \\ \hline -631 &= C \end{aligned}$$

c. How many cubic feet of snow are on the driveway at 9 A.M.?

$$\int_0^9 7te^{\cos t} dt - [108(9) - 631]$$

Falling - Removed

h(9)