

What you'll Learn About

- How to integrate a product by that cannot be done by recognition

Proof of Integration by Parts

1. Find $\frac{d}{dx}(uv) =$

$\frac{d}{dx}(uv) = u dv + v du$

2. Integrate both sides

$\int \frac{d}{dx}(uv) = \int u dv + \int v du$

$\int \frac{d}{dx}(uv) = \int u dv + \int v du$

$uv = \int u dv + \int v du$

3. Solve for $\int u dv$

$\int u dv = uv - \int v du$

Left - Derivative

Right - Antiderivative

Use ultra violet minus super vdu to integrate the following

2. $\int x e^x = x e^x - \int e^x dx = x e^x - e^x + C$

$\int u dv = uv - \int v du$

$\int x e^x = x e^x - e^x + C$

Use tabular integration to integrate the following

2. $\int x e^x =$

$\int x e^x dx = x e^x - \int e^x (1) dx = x e^x - e^x + C$

Use tabular integration to integrate the following

$$\begin{aligned}
 6. \int x^2 e^{-x} &= -x^2 e^{-x} + \int 2x e^{-x} dx \\
 &= -x^2 e^{-x} - 2x e^{-x} + \int 2e^{-x} dx \\
 &= \boxed{-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C}
 \end{aligned}$$

$$\begin{aligned}
 8. \int x^2 \cos\left(\frac{x}{2}\right) &= 2x^2 \sin\left(\frac{1}{2}x\right) - \int 4x \sin\left(\frac{1}{2}x\right) dx \\
 &= 2x^2 \sin\left(\frac{1}{2}x\right) - \left[-8x \cos\left(\frac{1}{2}x\right) + \int 8 \cos\left(\frac{1}{2}x\right) dx\right] \\
 &= \boxed{2x^2 \sin\left(\frac{1}{2}x\right) + 8x \cos\left(\frac{1}{2}x\right) - 16 \sin\left(\frac{1}{2}x\right) + C}
 \end{aligned}$$

Solve the initial value problem using tabular integration

~~Find the general solution~~ Solve the initial value problem

$$11) \frac{dy}{dx} = (x+2) \sin x \quad y=2 \text{ and } x=0$$

$$y = -(x+2) \cos x + \int \cos x$$

$$y = -(x+2) \cos x + \sin x + C$$

$$2 = -(2) + C$$

$$4 = C$$

$$y = -(x+2) \cos x + \sin x + 4$$

$$16. \frac{dy}{dx} = 2x\sqrt{x+2} \quad y(-1) = 0$$

$$\frac{dy}{dx} = \frac{2x(x+2)^{1/2}}{\frac{2}{3}(x+2)^{3/2}}$$

$$y = \frac{4x}{3}(x+2)^{3/2} - \int \frac{4}{3}(x+2)^{3/2}$$

$$y = \frac{4x}{3}(x+2)^{3/2} - \frac{4 \cdot 2}{3 \cdot 5/2}(x+2)^{5/2} + C$$

$$0 = -\frac{4}{3} - \frac{8}{15} + C$$

$$y = \frac{4x}{3}(x+2)^{3/2} - \frac{8}{15}(x+2)^{5/2} + \frac{28}{15}$$

$$-\frac{4(5)}{3(5)} - \frac{8}{15}$$

$$-\frac{20}{15} - \frac{8}{15}$$

$$0 = -\frac{28}{15} + C$$

Use tabular integration to integrate the following

$$10. \int x^2 \ln x dx$$

Use ultra violet minus super vdu to integrate the following

$$10. \int x^2 \ln x dx$$

Use tabular integration to integrate the following

A. $\int \arcsin(x) dx$

19. $\int e^x \cos(2x) dx$

Top Heavy Integrals

$$A. \int \frac{x^2 + x}{x} dx$$

$$B. \int \frac{\sqrt{x} + 5}{x} dx$$

$$C. \int \frac{x^3 + 2x}{\sqrt{x}} dx$$