What you'll Learn About

- How integrate by separating the variables

$A)\ (dx)\ \frac{dy}{dx} = x + 2\ dx$

$\int 1 dy = \int (x+2) dx$

$y = \frac{1}{2}x^2 + 2x + C$

$e^{x+C} = e^x e^C$

$= Ae^x$

$B)\ (dx)\ \frac{dy}{dx} = (y+2)\ dx$

$\frac{dy}{(y+2)} = \frac{(y+2)dx}{(y+2)}$

$\int \left(\frac{1}{y+2}\right) dy = \int 1 dx$

$\ln(y+2) = x + C$

$y + 2 = e^{x+C}$

$\boxed{y = e^{x+C} - 2}$

$\boxed{y = Ae^x - 2}$

$C)\ \cancel{(dx)}\ \frac{dy}{\cancel{dx}} = \frac{5x}{y}\ (dx)$ when x = 1 and y = 2

$y\,dy = \frac{5x\,dx}{\cancel{y}}\ \cancel{(y)}$

$\int (y) dy = \int (5x) dx$

$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C$

$\frac{1}{2}(4) = \frac{5}{2} + C$

$-\frac{1}{2} = C$

$\left(\frac{1}{2}y^2 = \frac{5}{2}x^2 - \frac{1}{2}\right)2$

$y^2 = 5x^2 - 1$

$\boxed{y = \sqrt{5x^2 - 1}}$

$x^{1/2}$	D) $\frac{dy}{dx} = y\sqrt{x}$ when x = 1 and y = 2 $\frac{dy}{y} = \frac{\cancel{y}\sqrt{x}\,dx}{\cancel{y}}$ $\int \frac{1}{y}\,dy = \int \sqrt{x}\,dx$ $\ln\lvert y\rvert = \frac{2}{3}x^{3/2} + C$ $\ln\lvert 2\rvert = \frac{2}{3} + C$ $\ln 2 - \frac{2}{3} = C$ $\ln y = \frac{2}{3}x^{3/2} + \ln 2 - \frac{2}{3}$ $\boxed{y = e^{\frac{2}{3}x^{3/2} + \ln 2 - \frac{2}{3}}}$ $y = e^{\frac{2}{3}x^{3/2}} \cdot e^{\ln 2} \cdot e^{-2/3}$ $\boxed{y = 2e^{\frac{2}{3}x^{3/2} - 2/3}}$ E) $\frac{dy}{dx} = y\sqrt{x}$ when x = 1 and y = −2 $\frac{dy}{y} = \frac{y\sqrt{x}\,dx}{y}$ $\int \frac{1}{y}\,dy = \int x^{1/2}\,dx$ $\ln\lvert y\rvert = \frac{2}{3}x^{3/2} + C$ $\ln\lvert -2\rvert = \frac{2}{3} + C$ $\ln 2 - \frac{2}{3} = C$ $\ln\lvert y\rvert = \frac{2}{3}x^{3/2} + \ln 2 - \frac{2}{3}$ $\ln(-y) = \frac{2}{3}x^{3/2} + \ln 2 - \frac{2}{3}$ $-y = e^{\frac{2}{3}x^{3/2} + \ln 2 - \frac{2}{3}}$ $\boxed{y = -2e^{\frac{2}{3}x^{3/2} - 2/3}}$

	$E)\quad \frac{dy}{dx} = -yx - y \qquad f(-2) = 1$

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What you'll Learn About

- How to use tangent line approximations to estimate a functions value for a given value of x

1. Consider the differential equation $\frac{dy}{dx} = x - 1$. Let y = f(x) be the particular solution to the given differential equation with initial condition f(1) = 2. Use Euler's Method, starting at x = 1 with three steps of equal size, to approximate f(1.3). Show the work that leads to your answer.

$f(1) = 2$
$f(1.1) = 2$
$f(1.2) = 2.01$
$\boxed{f(1.3) = 2.03}$

$\boxed{\frac{dy}{dx} = x - 1}$

$(1, 2)$ $\frac{dy}{dx} = 0$ $y = 2 + 0(x-1)$ $y = 2 + 0(1.1 - 1)$

$(1.1, 2)$ $\frac{dy}{dx} = .1$ $\boxed{y = 2 + .1(x - 1.1)}$ $\boxed{y = 2 + .1(1.2 - 1.1)}$

$(1.2, 2.01)$ $\frac{dy}{dx} = .2$ $\boxed{y = 2.01 + .2(x - 1.2)}$ $y = 2.01 + .2(1.3 - 1.2)$

2. Consider the differential equation $\frac{dy}{dx} = x - 2y$. Let y = f(x) be the particular solution to the given differential equation with initial condition f(2) = 1. Use Euler's Method, starting at x = 2 with three steps of equal size, to approximate f(1.7). Show the work that leads to your answer.

$f(2) = 1$
$f(1.9) = 1$
$f(1.8) = 1.01$
$\boxed{f(1.7) = 1.032}$

$(2, 1)$ $\frac{dy}{dx} = 0$ $y = 1 + 0(x - 2)$ $y = 1$

$(1.9, 1)$ $\frac{dy}{dx} = -.1$ $y = 1 - .1(x - 1.9)$ $y = 1 - .1(1.8 - 1.9)$

$(1.8, 1.01)$ $\frac{dy}{dx} = -.22$ $y = 1.01 - .22(x - 1.8)$ $y = 1.01 - .22(1.7 - 1.8)$

$y = 1.01 + .022$

$1.8 - 2.02$