

Top Heavy Integrals

$$A. \int \frac{x^2 + x}{x} dx$$

$$B. \int \frac{\sqrt{x} + 5}{x} dx$$

$$C. \int \frac{x^3 + 2x}{\sqrt{x}} dx$$

$$2x+1=0$$

$$\frac{-1}{2} = -\frac{1}{2}$$

$$x = -\frac{1}{2}$$

$$C) \int \frac{2}{2x^2+3x+1} dx = \int \frac{4}{2x+1} + \frac{-2}{x+1}$$

$$\int \frac{2}{(2x+1)(x+1)}$$

$$= \frac{1}{2} \cdot 4 \ln|2x+1| - 2 \ln|x+1| + C$$

$$\frac{2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$$

$$\ln|(2x+1)^2| - \ln(x+1)^2 + C$$

$$2 = A(x+1) + B(2x+1)$$

$$\ln \left| \frac{(2x+1)^2}{(x+1)^2} \right| + C$$

$$x=-1 \quad 2 = -B \quad x = -\frac{1}{2} \quad 2 = \frac{1}{2}A$$

$$-2 = B \quad 4 = A$$

$$D) \int \frac{x^3-5}{x^2-1} dx = \int x + \frac{x-5}{x^2-1} = \int x + \int \frac{x-5}{x^2-1}$$

$$x(x^2-1) \overline{\begin{array}{r} x \\ x^3+0x^2+0x-5 \\ -x^3 \quad +1x \\ \hline 1x-5 \end{array}}$$

$$\int x + \int \frac{3}{x+1} - \frac{2}{x-1}$$

$$\frac{1}{2}x^2 + 3 \ln|x+1| - 2 \ln|x-1| + C$$

$$\frac{x-5}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$x-5 = A(x-1) + B(x+1)$$

$$x=1 \quad -4 = 2B \quad x=-1 \quad -6 = -2A$$

$$-2 = B \quad 3 = A$$

Find $f(x)$

$$E) f'(x) = \frac{2x^3}{x^3 - x}$$

$$f(x) = \int 2 + \frac{2x}{x^3 - x}$$

$$2(x^3 - x) \overline{\begin{array}{r} 2 \\ \underline{2x^3} \\ -2x^3 + 2x \\ \hline 2x \end{array}}$$

$$\frac{2x}{x^3 - x} = \frac{2x}{x(x^2 - 1)} = \frac{2x}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$F) \int \frac{9}{x^2+1} = 9 \arctan(x) + C \quad G) \int \frac{9x}{x^2+1} = \frac{9}{2} \ln(x^2+1)$$

$$H) \int \frac{9x}{x^2-1} = \frac{9}{2} \ln(x^2-1) + C$$

$$I) \int \frac{9}{x^2-1} \quad \text{Partial Fractions}$$