	Top Heavy Integrals $A.\ \int \frac{x^2 + x}{x}\,dx$ $B.\ \int \frac{\sqrt{x} + 5}{x}\,dx$ $C.\ \int \frac{x^3 + 2x}{\sqrt{x}}\,dx$

8.4 Improper Integrals

What you'll Learn About

- How to integrate functions that approach infinity or functions that approach an asymptote

2) $\int_1^\infty \frac{dx}{x^{1/3}} = \int_1^\infty x^{-1/3}dx = \frac{3}{2}x^{2/3}\Big]_1^\infty$

$\lim_{b\to\infty}\left[\frac{3}{2}x^{2/3}\right]_1^b = \lim_{b\to\infty}\left[\frac{3}{2}b^{2/3} - \frac{3}{2}\right] = \infty$

Area Diverges

6) $\int_1^\infty \frac{2dx}{x^3} = \int_1^\infty 2x^{-3}dx = -x^{-2}\Big]_1^\infty$

$\lim_{b\to\infty}\left[-\frac{1}{x^2}\right]_1^b = \lim_{b\to\infty}\left[\cancel{\frac{-1}{b^2}} + 1\right] = 1$

0

Area Converges to 1

10) $\int_{-\infty}^{0} \frac{dx}{(x-2)^3} = \int_{-\infty}^{0} (x-2)^{-3}\,dx = -\frac{1}{2}(x-2)^{-2}\Big]_{-\infty}^{0}$

$\lim_{b\to-\infty}\left[\frac{-1}{2(x-2)^2}\right]_b^0 = \lim_{b\to\infty}\left[-\frac{1}{8} - \left(\cancel{\frac{-1}{2(b-2)^2}}\right)\right] = -\frac{1}{8}$

0

14) $\int_{-\infty}^{0} \frac{2dx}{x^2-4x+3} = \int_{-\infty}^{0} \frac{1}{x-3} - \frac{1}{x-1} = \ln|x-3| - \ln|x-1|\Big]_{-\infty}^{0}$

$\frac{2}{(x-3)(x-1)} = \frac{A}{(x-3)} + \frac{B}{(x-1)}$

$2 = A(x-1) + B(x-3)$

$x=1$ $\quad 2=-2B \quad -1=B$

$x=3$ $\quad 2=2A \quad 1=A$

$\lim_{b\to\infty}\left[\ln\left|\frac{x-3}{x-1}\right|\right]_b^0 = \lim_{b\to\infty}\left[\ln 3 - \ln\left|\frac{b-3}{b-1}\right|\right]$

1

$\lim_{b\to-\infty}\left[\ln 3 - \ln 1\right]$

$\boxed{\ln 3}$

x-intercept $(y=0)$ $0=\frac{\ln x}{x^2}$ $0=\ln x$	18) $\int_{-\infty}^{0} x^2 e^x dx = x^2e^x - \int 2xe^x$ $= x^2e^x - [2xe^x - \int 2e^x]$ $= x^2e^x - 2xe^x + 2e^x \Big]_{-\infty}^{0}$ $\lim_{b\to-\infty}\left[x^2e^x-2xe^x+2e^x\right]_{b}^{0} = \lim_{b\to-\infty}\left[2-(\cancel{b^2e^b}-\cancel{2be^b}+\cancel{2e^b})\right]$ 0 0 0 $\boxed{2}$ 43. Find the area of the region in the first quadrant that lies under the given curve $y=\frac{\ln x}{x^2}$ $\int_1^{\infty}\frac{\ln x}{x^2} = \int_1^{\infty} \ln x (x^{-2}) dx = -\frac{1}{x}\ln x + \int \frac{1}{x^2} dx$ ($\frac{1}{x}$ | $-x^{-1} = -\frac{1}{x}$) $= -\frac{1}{x}\ln x + \int x^{-2} dx$ $= -\frac{1}{x}\ln x - \frac{1}{x}\Big]_1^{\infty}$ $\lim_{b\to\infty}\left[\frac{-1}{x}\ln x - \frac{1}{x}\right]_1^b = \left(\cancel{-\frac{1}{b}\ln b} - \cancel{\frac{1}{b}}\right) - (0-1) = 1$ 0 0

$-\frac{\ln b}{b}$