Chapter 5 AP problems

Riemann Sums with Average Value

2014 BC4

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/min)	0	100	40	-120	-150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

   c) At time t = 12, train A's position is defined as $300+\int_2^{12} v_a(t)dt$ Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time t = 12.

2013 BC3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.5

Use a midpoint sum with three subinterval of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)dt$ in the context of the problem.

2011 #2

t(minutes)	0	2	5	9	10
H(t) degrees C	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature H(t) is measured in degrees Celsius. Values of H(t) at selected values of time t are shown in the table above

Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of the problem. Use a trapezoidal sum with four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

2010 #2

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (t=0) and 8 P.M. (t=8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of E(T), in hundreds of entries, at various times t are shown in the table.

t(hours)	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

Use a trapezoidal sum with four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^{8} E(t)\,dt$ in terms of number of entries.

2012 #1

t(minutes)	0	4	9	15	20
W(t) degrees F	55.0	57.1	61.8	67.9	71.0

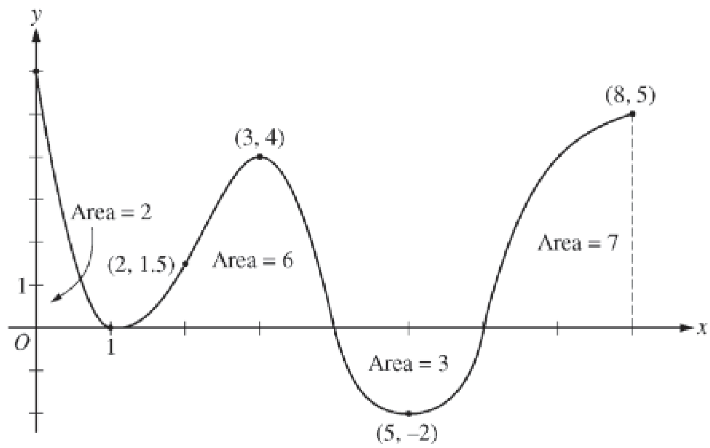
The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time t = 0, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time t = 0. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

FFTOC using graphs

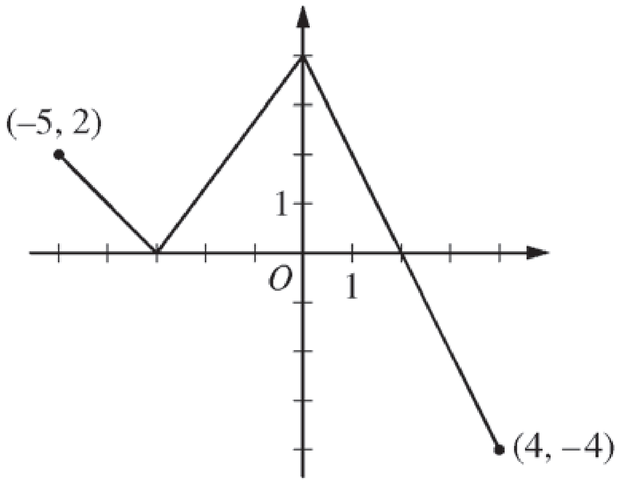
2013BC4

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at x = 1, x = 3, and x = 5. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies f(8) = 4.



Graph of f'

a) For all values of x on the open interval 0 < x < 8 for which the function f has a local minimum. Justify your answer.
b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.
c) On what intervals contained in 0 < x < 8 is the graph of f both concave down and increasing? Explain your reasoning.
d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = \frac{-5}{2}$, find the slope of the line tangent to the graph of g at x = 3.



Graph of f

3. The function f is defined on the closed interval [-5 , 4]. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)dt$.

a) Find g(3)

b) On what open intervals contained in -5 < x < 4 is the graph of g both increasing and concave down? Give a reason for your answer.

c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

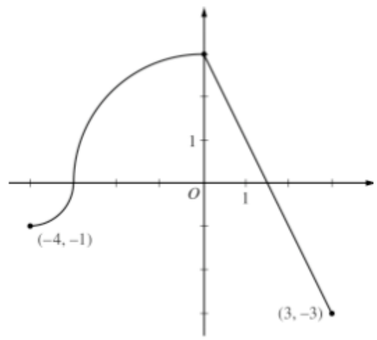
d) The function p is defined by $p(x) = f(x^2 - x)$. find the slope of the line tangent to the graph of p at the point where x = -1.

Extrema

2011 #4
The continuous function f is defined on the interval $-4 \le t \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure

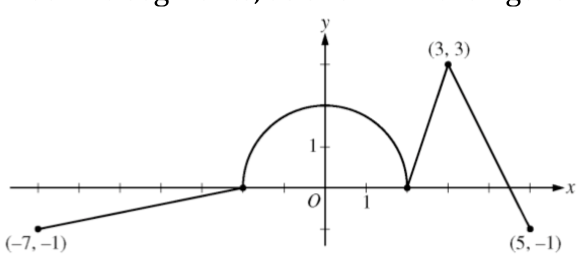
Let $g(x) = 2x + \int_0^x f(t)d$

Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le t \le 3$. Justify your answer.



Graph of f

2010 #5
The function g is defined and differentiable on the closed interval [-7,5] and satisfies g(0) = 5. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure.

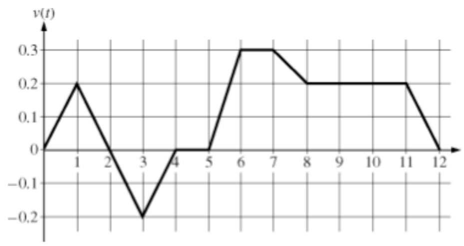


Graph of g'

The function h is defined by h(x) = g(x) - .5x^2. Find the x-coordinate of each critical point of h, where -7 < x < 5, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

2009 #1

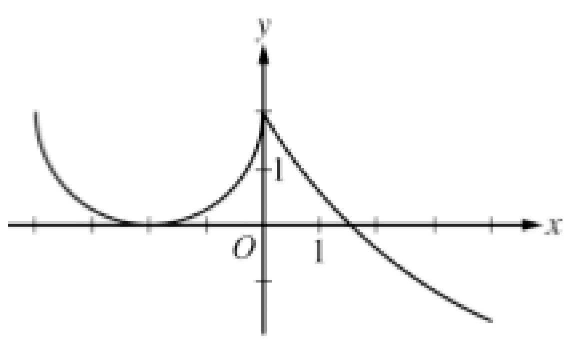
Caren rides her bicycle along a straight road from home to school, starting at home at time t = 0 minutes and arriving at school at time t = 12 minutes. During the time interval $0 \le t \le 12$ minutes, her velocity, v(t), in miles per minute is modeled by the piecewise–linear function whose graph is shown.



Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

<u>2009 #6</u>

The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}$. The graph of the continuous function f', shown, has x-intercepts at x = -2 and x = $3\ln\frac{5}{3}$. The graph of g on $-4 \le x \le 0$ is a semicircle and f(0) = 5.



Graph of f'

For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.