

Determine the limit algebraically, if it exists.

$$1) \lim_{x \rightarrow 4} \frac{x^2 + 4x - 32}{x^2 - 16}$$

$$2) \lim_{x \rightarrow 0} \frac{\frac{1}{x+6} - \frac{1}{6}}{x}$$

$$3) \lim_{x \rightarrow 4^-} \frac{|x-4|}{x-4}$$

Find the indicated limit.

4) a) $\lim_{x \rightarrow 0^+} \frac{7x}{|x|}$

b) $\lim_{x \rightarrow 0^-} \frac{7x}{|x|}$

c) $\lim_{x \rightarrow 0} \frac{7x}{|x|}$

Evaluate or determine that the limit does not exist for each of the limits for the given function f.

5)

$$f(x) = \begin{cases} -2x - 2, & \text{for } x < 1, \\ 1, & \text{for } x = 1, \\ -4x + 8, & \text{for } x > 1 \end{cases}$$

(a) $\lim_{x \rightarrow 1^-} f(x)$

(b) $\lim_{x \rightarrow 1^+} f(x)$

(c) $\lim_{x \rightarrow 1} f(x)$

Find the limits of $f(x)$

$$6) f(x) = \begin{cases} \frac{x-4}{x-2}, & x \leq 0 \\ \frac{1}{x^2}, & x > 0 \end{cases}$$

(a) $\lim_{x \rightarrow \infty} f(x)$

(b) $\lim_{x \rightarrow 0} f(x)$

(c) $\lim_{x \rightarrow 0^-} f(x)$

(d) $\lim_{x \rightarrow 0^+} f(x)$

Find the points of discontinuity. Identify each type of discontinuity. Use limits to defend your choice.

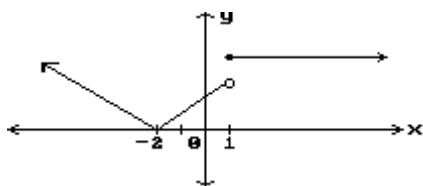
$$7) y = \frac{x+4}{x^2 - 14x + 48}$$

For problems 8 and 9 find all points where the function is discontinuous. Then identify any x-values where the function is not differentiable. Be specific which points are discontinuous and/or which are not differentiable.

8)

$$f(x) = \begin{cases} \frac{x^2 - 16}{x + 4}, & x \neq -4 \\ 10, & x = -4 \end{cases}$$

9)



Find a value for a so that the function f(x) is continuous.

$$10) f(x) = \begin{cases} x^2 - 5, & x < 4 \\ 5ax, & x \geq 4 \end{cases}$$

11) Sketch a graph of a function that satisfies the given conditions.

$$\lim_{x \rightarrow 2} f(x) = 2$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\lim_{x \rightarrow 2^+} f(x) = \infty$$

$$\lim_{x \rightarrow 2^-} f(x) = \infty$$

12) Sketch a graph of a function that satisfies the given conditions.

$$\lim_{x \rightarrow 5} f(x) \text{ does not exist} \quad \text{and} \quad \lim_{x \rightarrow 5^+} f(x) = f(5) = 2$$

Use a definition of the derivative to set up the limit you would use to find the slope at the given point. Then, using the substitution $h = x - a$, set up the limit in terms of h , that would find the derivative of $f(x)$ at $x = 4$.

13) $f(x) = \frac{4}{x+3}$ at $x = 4$

Use a definition of the derivative to set up the limit you would use to find the slope at the given point. Then, using the substitution $h = x - a$, set up the limit, in terms of h , that would find the derivative of $f(x)$ at $x = -3$. After you have set up the limit, for extra credit find the actual slope at the point $(-3, -54)$ by evaluating a definition of the derivative.

14) $f(x) = 3x - 5x^2$ at the point $(-3, -54)$.

15)

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/min)	0	100	40	-120	-150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

b) Do the data in the table support the conclusion that train A's velocity is 50 meters per minute some time t with $0 < t < 2$? Give a reason for your answer.