

49. An object moving along a curve in the xy-plane has position  $(x(t), y(t))$  at time  $t \geq 0$  with  $\frac{dx}{dt} = 2 + \sin(t^2)$ . The derivative  $\frac{dy}{dt}$  is not explicitly given. at time  $t = 2$ , the object is at position  $(3, 5)$ .

a) Find the x-coordinate of the position of the object at time  $t = 4$ .

$$x(4) = \underset{\substack{\uparrow \\ \text{start}}}{3} + \int_2^4 \underset{\substack{\uparrow \\ \text{displacement}}}{2 + \sin(t^2)} dt = 6.942$$

b) At time  $t = 2$ , the value of  $\frac{dy}{dt} = -6$ . Write an equation for the line tangent to the curve at the point  $(x(2), y(2))$ .  $(3, 5)$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \quad \frac{dy}{dx} = \frac{-6}{2 + \sin(2^2)}$$

$$y = y_1 + m(x - x_1)$$

$$y = 5 + \frac{-6}{2 + \sin 4} (x - 3)$$

c) Find the speed of the object at time  $t = 2$ .

$$\text{speed} = |v(t)| = \sqrt{(v_x)^2 + (v_y)^2}$$

$$= \sqrt{(2 + \sin 4)^2 + (-6)^2}$$

d) For  $t \geq 3$ , the line tangent to the curve at  $(x(t), y(t))$  has a slope of  $2t - 1$ . Find the acceleration vector of the object at time  $t = 4$ .

$$\frac{dy}{dt} = \frac{dx}{dt} \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = 2t - 1$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$2t - 1 = \frac{\left(\frac{dy}{dt}\right)}{\left(2 + \sin(t^2)\right)}$$

$$v(t) = \left\langle 2 + \sin(t^2), (2 + \sin(t^2))(2t - 1) \right\rangle$$

$$a(t) = \left\langle 2t \cos(t^2), (2 + \sin(t^2)) \cdot 2 + (2t - 1) 2t \cos(t^2) \right\rangle$$

$$a(4) = \left\langle 8 \cos 16, 2(2 + \sin 16) + 56 \cos 16 \right\rangle$$

$$(2 + \sin(t^2))(2t - 1) = dy/dt$$

2012 #2

For  $t \geq 0$ , a particle is moving along a curve so that its position at time  $t$  is  $(x(t), y(t))$ .

At time  $t = 2$ , the particle is at position  $(1, 5)$ . It is known that

$$\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t} \quad \text{and} \quad \frac{dy}{dt} = \sin^2 t.$$

Find the speed of the particle at time  $t = 4$ .

Find the slope of the path of the particle at time  $t = 2$ .

Is the horizontal movement of the particle to the left or to the right at time  $t = 2$ ?

Explain your answer.

Find the  $x$ -coordinate of the particle's position at time  $t = 4$ .

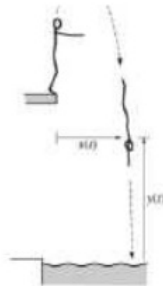
Find the acceleration vector of the particle at time  $t = 4$ .

Find the distance traveled by the particle from time  $t = 2$  to  $t = 4$ .

2009 #3

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time  $t$  seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by  $x(t)$ , and the vertical distance from the water surface to her shoulders is given by  $y(t)$ , where  $x(t)$  and  $y(t)$  are measured in meters.

Suppose that the divers shoulders are 11.4 meters above the water when she makes her leap and that  $\frac{dx}{dt} = .8$  and  $\frac{dy}{dt} = 3.6 - 9.8t$ , for  $0 \leq t \leq A$ , where  $A$  is the time that the divers shoulders enter the water.



Find how long it takes the diver to achieve maximum vertical distance from the water surface. What is the diver's maximum vertical distance from the water surface to the diver's shoulders?

Find  $A$ , the time that the diver's shoulders enter the water.

Find the total distance traveled by the diver's shoulders from the time they leap from the platform until the time her shoulders enter the water.

Find the angle  $\theta$ ,  $0 < \theta < \frac{\pi}{2}$ , between the path of the diver and the water at the instant the diver's shoulders enter the water