10.2: Vectors

What you'll Learn About

- Calculus using Vectors

A particle moves in an elliptical path so that its position at any time $t \geq 0$ is given by $\langle 4sint, 2\,cost \rangle$

a) Find the velocity and acceleration vectors.

$x = 4\sin t$

$y = 2\cos t$

$s(t) = \langle 4\sin t, 2\cos t \rangle$

$v(t) = \langle 4\cos t, -2\sin t \rangle$

$a(t) = \langle -4\sin t, -2\cos t \rangle$

b) Find the velocity, acceleration, and speed vectors at $t = \frac{\pi}{4}$.

$v\left(\frac{\pi}{4}\right) = \left\langle 4\cos\frac{\pi}{4}, -2\sin\frac{\pi}{4} \right\rangle = \left\langle \frac{4\sqrt{2}}{2}, \frac{-2\sqrt{2}}{2} \right\rangle = \langle 2\sqrt{2}, -\sqrt{2} \rangle$

$= \langle 2.818, -1.414 \rangle$

$a\left(\frac{\pi}{4}\right) = \left\langle -4\sin\frac{\pi}{4}, -2\cos\frac{\pi}{4} \right\rangle = \left\langle \frac{-4\sqrt{2}}{2}, \frac{-2\sqrt{2}}{2} \right\rangle = \langle -2\sqrt{2}, -\sqrt{2} \rangle$

speed = length (magnitude) $= |v(t)| = \sqrt{(2\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{8+2} = \sqrt{10}$

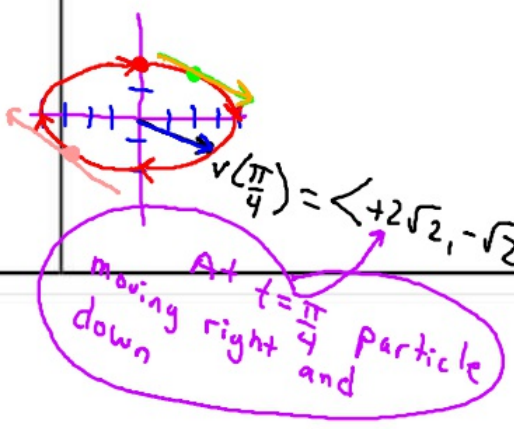
Pythagorean thm

$\sqrt{v_x^2 + v_y^2}$

c) Sketch the path of the particle and show the velocity vector at $t = \frac{\pi}{4}$

$x = 4\sin t \quad y = 2\cos t$

t	x	y
0	0	2
$\pi/2$	4	0



$v(t) = \left\langle \frac{1}{t+1}, (t+2)^{-2} \right\rangle$ $(3, -2)$

The velocity v(t) of a particle moving in the plane is given, along with the position of the particle at time t = 0.

a) Find the position of the particle at time t = 3.

position at $t=0$

$$x = 3 + \int_0^3 \frac{1}{t+1}$$

$$x = 3 + \left[\ln|t+1|\right]_0^3$$

$$x = 3 + \left[\ln 4 - \ln 1\right]$$

$$x = 3 + \ln 4$$

$$y = -2 + \int_0^3 (t+2)^{-2}$$

$$y = -2 + \left[-(t+2)^{-1}\right]_0^3$$

$$y = -2 + \left[\frac{-1}{t+2}\right]_0^3$$

$$y = -2 + \left[-\frac{1}{5} - \left(-\frac{1}{2}\right)\right]$$

$$y = -2 + \left[-\frac{1}{5} + \frac{1}{2}\right]$$

Position at $t=3$ $\left\langle 3 + \ln 4, \frac{-17}{10} \right\rangle$

b) Find the distance traveled from t = 0 to t = 3.

Arc Length $= \int \sqrt{(v_x)^2 + (v_y)^2}$

$$= \int_0^3 \sqrt{\left(\frac{1}{t+1}\right)^2 + \left((t+2)^{-2}\right)^2}$$

$$\approx 1.418$$