10.1: Parametric Equations

What you'll Learn About

- Calculus using Parametric Equations

Convert the following parametric equations into a Cartesian Equation. Then find the first derivative and 2nd derivative.

$y =$

A) $x = 4t$ $y = t^2$ at $t = 1$

$\frac{x}{4} = t$

$y = \left(\frac{x}{4}\right)^2$

$y = \frac{1}{16}x^2$ $\quad y' = \frac{1}{8}x$ $\quad y'' = \frac{1}{8}$

$y'(4) = \frac{1}{8}(4) = \frac{1}{2}$

$t = 1$

Find the first derivative and 2nd derivative of the parametric curve in terms of t.

$y = (t)^2$

B) $x = 4t$ $y = t^2$ at $t = 1$

$\frac{dx}{dt} = 4$ $\quad \frac{dy}{dt} = 2t$

$\frac{dt}{dx} = \frac{1}{4}$ ←

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2t}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{dy}{dx} = \frac{2t}{4} = \frac{1}{2}t$

$\frac{d^2y}{dx^2} = \frac{1}{2}\left(\frac{dt}{dx}\right) = \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$

Find the first derivative and 2nd derivative of the parametric curve in terms of t

16. $x = \ln(5t)$ $y = e^{5t}$

$\frac{dx}{dt} = \frac{1}{5t} \cdot 5 = \frac{1}{t}$ $\frac{dy}{dt} = e^{5t} \cdot 5$

$\frac{dt}{dx} = t$

$\frac{dy}{dx} = \frac{5e^{5t}}{\left(\frac{1}{t}\right)} = (5t)(e^{5t})$

$\frac{d^2y}{dx^2} = 5t\left[e^{5t} \cdot 5\frac{dt}{dx}\right] + e^{5t}\left[5\frac{dt}{dx}\right]$

$= 5t\left[e^{5t} \cdot 5t\right] + e^{5t} \cdot 5t$

Abs Max/Min

(1) Take derivative

(2) set derivative = 0 - critical pts

(3) Plug C.P and endpoints into original

Determine the leftmost point on the parametric curve between [-2, 3]

18. $x = t^2 + 2t$ $y = t^2 - 2t + 3$

$\frac{dx}{dt} = 2t + 2$

$0 = 2t + 2$

$-1 = t$

$x(-2) = 0$

$x(-1) = -1$ Left most

$x(3) = 15$

tangent line vertical

$\frac{dy}{dx}$ undefined

$\frac{dx}{dt} = 0$

tangent line horizontal

$\frac{dy}{dx} = 0$

$\frac{dy}{dt} = 0$

26. Find the points at which the tangent line to the curve is horizontal or vertical.

$x = -2 + 3\cos t \qquad y = 1 + 3\sin t \qquad [0, 2\pi]$

$\frac{dx}{dt} = -3\sin t \qquad \frac{dy}{dt} = 3\cos t$

$0 = -3\sin t \qquad 0 = 3\cos t$

$0 = \sin t \qquad 0 = \cos t$

$t = 0, \pi, 2\pi \qquad t = \pi/2, 3\pi/2$

Vertical tangents — Horizontal Tangents

28. Find the length of the curve.

$x = 3\sin t \quad y = \cos t \quad [0, \pi]$

$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -\sin t$

Parametric

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$L = \int_0^{\pi} \sqrt{(3\cos t)^2 + (-\sin t)^2}$$

Cartesian Arc Length

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$