

Determine the intervals of concavity and the inflection points

A) $f(x) = x^{2/5}$

$$f'(x) = \frac{2}{5} x^{-3/5}$$

$$f''(x) = -\frac{6}{25} x^{-8/5}$$

$$0 = \frac{-6}{25(x)^{8/5}}$$

$$\begin{array}{l} f'' = 0 \\ \hline -6 \neq 0 \end{array} \left\{ \begin{array}{l} f'' \text{ und} \\ \hline x = 0 \\ \uparrow \\ \text{P.I.P.S} \end{array} \right.$$

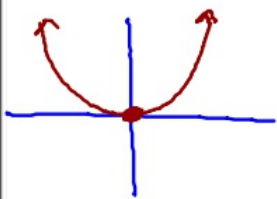
$$f''(-1) = \frac{-6}{25(-1)^{8/5}} < 0$$

$f(x)$ concave down $(-\infty, 0)$

$$f''(1) = \frac{-6}{25} < 0$$

$f(x)$ concave down $(0, \infty)$

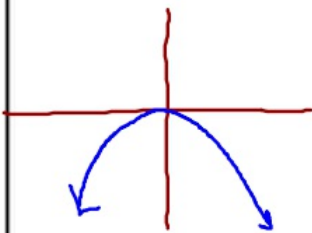
Local Extrema and the 2nd Derivative



Determine the local extrema using the second derivative test

A) $y = x^2$

$y' = 2x$ $y'' = 2 > 0$ y concave up
 C.P. $2x = 0$
 $x = 0$ Local min b/c $y''(0) > 0$



B) $y = -x^2$

$y' = -2x$ $y'' = -2 < 0$ y concave down
 C.P. $-2x = 0$
 $x = 0$ Local Max b/c $y''(0) < 0$

At a C.P. if $f(x)$ concave down local max

At a C.P. if $f(x)$ concave up local min

Determine the local extrema using the second derivative test

25) $f(x) = x^3 - 12x^2 + 45x$

$f'(x) = 3x^2 - 24x + 45$

$f''(x) = 6x - 24$

$0 = 3x^2 - 24x + 45$

$f''(3) = -6 < 0$

$0 = x^2 - 8x + 15$

f concave down $x=3$ local max

$0 = (x-5)(x-3)$

$x=5$ $x=3$

$f''(5) = 6 > 0$

f concave up $x=5$ local min

- ① Find C.P.
- ② Plug C.P. into 2nd derivative

27) $f(x) = 3x^4 - 8x^3 + 6x^2$

$f'(x) = 12x^3 - 24x^2 + 12x$

$f''(x) = 36x^2 - 48x + 12$

$0 = 12x(x^2 - 2x + 1)$

$f''(0) = +12 > 0$

$0 = 12x(x-1)(x-1)$

f concave up $x=0$ local min

C.P. $x=0$ $x=1$

$f''(1) = 0$ $x=1$ P.I.P's

local min

$f'(\frac{1}{2}) = 6(-\frac{1}{2})(-\frac{1}{2}) > 0$ f inc $(0,1)$
 $f'(2) = 24(1)(1) > 0$ f inc $(1,\infty)$

Determine the local extrema using the second derivative test

31) $f(x) = 6x^{3/2} - 4x^{1/2}$

5) $f(x) = 10x^3 - x^5$

What you'll Learn About
 How to describe the key features of a graph using the 1st and 2nd derivative

Absolute Extrema

$$f(-2) = 16 + 24 - 3 = 37$$

$$f(0) = -3$$

$$f(2) = -16 + 24 - 3 = 5$$

$$f(5) = -250 + 150 - 3 = -103$$

$$f(-2) = 37$$

Abs Max

$$f(5) = -103$$

Abs Min

Critical Points

$$2) f(x) = -2x^3 + 6x^2 - 3$$

$[-2, 5]$

$$f'(x) = -6x^2 + 12x$$

$$0 = -6x^2 + 12x$$

$$0 = -6x(x-2)$$

$$x = 0$$

$$x = 2$$

$$f'(-1) = -18 < 0$$

f dec $(-\infty, 0)$

$$f'(1) = 6 > 0$$

f inc $(0, 2)$

$$f'(3) = -18 < 0$$

f dec $(2, \infty)$

$x = 0$ local min b/c

f' changes from neg to pos

$x = 2$ local max b/c

f' changes from pos to neg

$$f''(x) = -12x + 12$$

$$0 = -12x + 12$$

$$x = 1$$

$$f''(0) = 12 > 0$$

f concave up $(-\infty, 1)$

$x = 0$ local min

$$f''(2) = -12 < 0$$

f concave down $(1, \infty)$

$x = 2$ local max