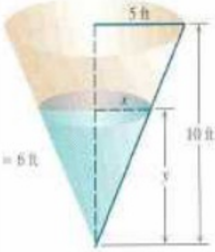
Particle Motion Summary Given the **Velocity v(t)** graph

Determine when the particle	Justify/Explain/Give a reason	Where to look on the velocity graph
Forward/Up/Right	$v(t) > 0$	Above the x-axis
Backward/Down/Left	$v(t) < 0$	Below the x-axis
Stopped/At rest	$v(t) = 0$	Touches x-axis
Changes Direction	$v(t) = 0$ and $v(t)$ changes sign	Crosses x-axis
Acceleration Positive	$v'(t) > 0$	Positive slope/Increasing
Acceleration Negative	$v'(t) < 0$	Negative slope/Decreasing
Acceleration Zero	$v'(t) = 0$	Zero slope/Constant
Acceleration Undefined	$v'(t)$ undefined	Corners/Cusps/Vertical Tangents
Speed increasing Speeding up	$v(t)$ and $a(t)$ have the same sign	Graph moving away from the x-axis
Speed decreasing	$v(t)$ and $a(t)$ have opposite signs	Graph moving toward the x-axis
Greatest Speed	$\lvert v(t) \rvert$ greatest	Graph farthest from x-axis in either direction

	Consider the curve defined by the equation $2y^3 + 6x^2y - 12x^2 + 6y = 1$ with $\frac{dy}{dx} = \frac{4x-2xy}{x^2+y^2+1}$ b) Write an equation of each horizontal tangent to the curve c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x and y-coordinates of P. d) Find $\frac{d^2y}{dx^2}$ in terms of y.

	C) Truck A travels east at 40 mi/hr. Truck B travels north at 30 mi/hr. How fast is the distance between the trucks changing 6 minutes later?
	D) Water runs into a conical tank at the rate of 9 ft³/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



$\boxed{\frac{dV}{dt} = 9\ ft^3/min \quad \text{Find } \frac{dh}{dt} \text{ when } h = 6\ ft}$

$\frac{h}{r} = \frac{10}{5}$

$\frac{h}{r} = \frac{2}{1}$

$2r = h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi\left[r^2\frac{dh}{dt} + h\left(2r\frac{dr}{dt}\right)\right]$

$9 = \frac{1}{3}\pi\left[r^2\frac{dh}{dt} + 6\left(2r\frac{dr}{dt}\right)\right]$

$9 = \frac{1}{3}\pi\left[3^2\frac{dh}{dt} + 6(2)(3)\frac{1}{2}\frac{dh}{dt}\right]$

$h = 2r$

$\frac{dh}{dt} = 2\frac{dr}{dt}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right) h$

$\boxed{V = \frac{1}{12}\pi h^3}$

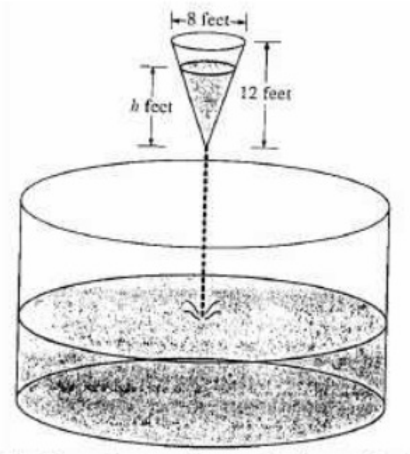
$\frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$

$9 = \frac{1}{4}\pi(6)^2\frac{dh}{dt}$

$9 = 9\pi\frac{dh}{dt}$

21. Water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth, h, in feet, of the water in the conical tank is changing at the rate of (h - 12) feet per minute. Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

$\frac{dh}{dt}$



$\frac{h}{r} = \frac{12}{4}$

$\frac{h}{r} = 3$

$h = 3r$

$\frac{h}{3} = r$

A) Write an expression for the volume of water in the conical tank as a function of h.

$V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2(h) \rightarrow \boxed{V = \frac{1}{27}\pi h^3}$

B) At what rate is the volume of water in the conical tank changing when h = 3? Indicate units of measure.

$\frac{dV}{dt} = \frac{1}{27}\pi\left(3h^2\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 (h-12)$

$\frac{dV}{dt} = \frac{1}{9}\pi(3)^2(3-12)\ ft^3/min = -9\pi\ ft^3/min$

C) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure.

Base Area is 400π

$\pi r^2 = 400\pi$

$V = (\pi r^2)h$

$V = 400\pi y$

$\cancel{y = h}$

$\cancel{\frac{dy}{dt} = \frac{dh}{dt}}$

$\frac{dV}{dt} = 400\pi\frac{dy}{dt}$

$\frac{+9\pi}{400\pi} = \frac{400\pi\frac{dy}{dt}}{400\pi}$

$\boxed{\frac{dy}{dt} = \frac{9}{400}\ ft/min}$