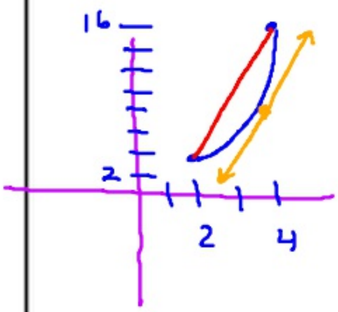
Use the Mean Value Theorem to determine where the slope of the secant line equals the slope of the tangent line



A) $f(x) = x^2$ [2,4]

$(2, 4)\ (4, 16)$

$f'(x) = 2x$

avg rate of change $= \frac{16-4}{4-2} = \frac{12}{2} = 6$

slope of secant $= 6$

$2x = 6$

$x = 3$

The slope of the secant equals the slope of the tangent because $f(x) = x^2$ is differentiable from [2,4]

Avg rate of change equals instantaneous rate of change

B) $f(x) = x^{\frac{1}{3}}$ [1,8]

$f'(x) = \frac{1}{3}x^{-2/3}$ $(1, 1)(8, 2)$

Avg Rate $= \frac{2-1}{8-1} = \frac{1}{7}$

$\frac{1}{3x^{2/3}} = \frac{1}{7}$ $\quad 7 = 3x^{2/3}$

$\left(\frac{7}{3}\right)^{3/2} = \left(x^{2/3}\right)^{3/2}$

$\left(\frac{7}{3}\right)^{3/2} = x$

C) $f(x) = x^{\frac{1}{3}}$ [0,1] [-1,1]

$f'(x) = \frac{1}{3x^{2/3}}$

UNDEFINED $x = 0$

MVT does not guarantee a pt. on [-1,1] because $f(x)$ not differentiable at $x=0$

(1) Avg Rate of Change

(2) Find $f'(x)$

(3) Set $f' =$ Avg rate

Rolle's Thm

D) $f(x) = x^2$ [-2,2]

$(-2, 4)(2, 4)$

$f'(x) = 2x$

Avg Rate $= \frac{4-4}{2-(-2)} = 0$

$2x = 0$

$x = 0$

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Let f be the function defined by f(x) = x + lnx. What is the value of c

Und at $x = 0$ →	which the instantaneous rate of change of f at x = c is the same as the average rate of change of f over [2, 6]?
$\frac{1}{x} = \frac{\ln 3}{4}$	$f'(x) = 1 + \frac{1}{x}$
$x = \frac{4}{\ln 3}$	$(2, 2+\ln 2)(6, 6+\ln 6)$
	Avg rate $= \frac{(6+\ln 6)-(2+\ln 2)}{6-2}$
	$= \frac{4+\ln 6 - \ln 2}{4}$
	$\frac{4+\ln 3}{4}$
	$1 + \frac{1}{x} = \frac{4+\ln 3}{4}$
	$-1 \qquad -1$
	$\frac{1}{x} = \frac{4+\ln 3}{4} - \frac{4}{4}$
	If $f(x) = \cos\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Find those values.
	$f'(x) = -\sin\left(\frac{x}{2}\right) \cdot \frac{1}{2}$
	$\left(\frac{\pi}{2}, \frac{\sqrt{2}}{2}\right)\left(\frac{3\pi}{2}, -\frac{\sqrt{2}}{2}\right)$
	avg rate $= \frac{-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{\frac{3\pi}{2} - \frac{\pi}{2}} = \frac{-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}}{\frac{\cancel{2}\pi}{\cancel{2}}} = \frac{-\sqrt{2}}{\pi}$
	$-\frac{1}{2}\sin\left(\frac{x}{2}\right) = -\frac{\sqrt{2}}{\pi}$
	$\frac{1}{2}\sin\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{\pi}$
	$\sin\left(\frac{x}{2}\right) = \frac{2\sqrt{2}}{\pi}$
	$\frac{x}{2} = \sin^{-1}\left(\frac{2\sqrt{2}}{\pi}\right)$
	$\frac{x}{2} = 1.120 \qquad x = 2.240$