

$$9 + (y+3)^2 = 25$$

$$(y+3)^2 = 16$$

$$y+3 = \pm 4$$

$$\begin{array}{r} -3 \\ -3 \end{array}$$

$$y = 4 - 3 = 1$$

$$y = -4 - 3 = -7$$

Determine the slope of the function at the given value of x  $x=1$

G)  $(x+2)^2 + (y+3)^2 = 25$

$$2(x+2) + 2(y+3) \cdot \frac{dy}{dx} = 0$$

$$2(y+3) \frac{dy}{dx} = -2(x+2)$$

$$\frac{dy}{dx} = \frac{-2(x+2)}{2(y+3)} = \frac{-(x+2)}{(y+3)}$$

$$\left. \frac{dy}{dx} \right|_{(1,1)} = \frac{-(1+2)}{1+3} = \frac{-3}{4}$$

$$\left. \frac{dy}{dx} \right|_{(1,-7)} = \frac{-(1+2)}{-7+3} = \frac{-3}{-4} = \frac{3}{4}$$

slope undefined  
 $y = -3$   
 $(, -3)$   $(, -3)$

Find where the slope of the curve is undefined

H)  $x^2 + 4xy + 4y^2 - 3x = 6$

$$2x + [4x \frac{dy}{dx} + y(4)] + 8y \frac{dy}{dx} - 3 = 0$$

denominator of slope = 0

$$4x \frac{dy}{dx} + 8y \frac{dy}{dx} = -2x - 4y + 3$$

$$\frac{\frac{dy}{dx} (4x + 8y)}{(4x + 8y)} = \frac{-2x - 4y + 3}{4x + 8y}$$

$$\frac{dy}{dx} = \frac{-2x - 4y + 3}{4x + 8y}$$

slope und?  
 $4x + 8y = 0$   
 $\frac{8y}{8} = \frac{-4x}{8}$   
 $y = -\frac{1}{2}x$   
 slope defined

$$x = -2y \leftarrow y = -\frac{1}{2}x$$

Find the lines that are tangent and normal to the curve at the given point

I)  $x^2 - \sqrt{3}xy + 2y^2 = 5$

$(\sqrt{3}, 2)$   
x y

$$2x - \left[ \sqrt{3}x \frac{dy}{dx} + y\sqrt{3} \right] + 4y \frac{dy}{dx} = 0$$

$$2\sqrt{3} - \left[ \sqrt{3} \cdot \sqrt{3} \frac{dy}{dx} + 2\sqrt{3} \right] + 4(2) \frac{dy}{dx} = 0$$

$$-3 \frac{dy}{dx} + 8 \frac{dy}{dx} = 0$$

$$5 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 0$$

$$y = 2 - 0(x - \sqrt{3})$$

$y = 2$  Tangent

$x = \sqrt{3}$  Normal

Find the lines that are tangent and normal to the curve at the given point

J)  $x \sin(2y) = y \cos(2x)$   $\left( \frac{\pi}{4}, \frac{\pi}{2} \right)$

$$x \cos(2y) \cdot 2 \frac{dy}{dx} + \sin(2y) = 2y(-\sin(2x)) + \cos(2x) \frac{dy}{dx}$$

$$\frac{\pi}{4} \cos\left(\frac{2\pi}{2}\right) \cdot 2 \frac{dy}{dx} + \sin\left(\frac{2\pi}{2}\right) = -2 \frac{\pi}{2} \sin\left(\frac{2\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right) \frac{dy}{dx}$$

$$\frac{\pi}{4} (-1) \cdot 2 \frac{dy}{dx} = -\pi$$

$$\frac{-2\pi}{4} \frac{dy}{dx} = -\pi \left( \frac{-4}{2\pi} \right)$$

$$\frac{dy}{dx} = 2$$

Tangent:  $y = \frac{\pi}{2} + 2(x - \frac{\pi}{4})$

Normal:  $y = \frac{\pi}{2} - \frac{1}{2}(x - \frac{\pi}{4})$

Determine the 2nd derivative of the function defined implicitly

K)  $2x^3 - 3y^2 = 8$

$$6x^2 - 6y \frac{dy}{dx} = 0$$

$$-6y \frac{dy}{dx} = -6x^2$$

$$\frac{dy}{dx} = \frac{-6x^2}{-6y}$$

$$\boxed{\frac{dy}{dx} = \frac{x^2}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{y(2x) - x^2 \left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{\cancel{y} 2xy - x^2 \left(\frac{x^2}{y}\right) \cancel{y}}{(y)y^2}$$

$$\boxed{= \frac{2xy^2 - x^4}{y^3}}$$

$$x^{1/3} - y^{1/3} = 1 \leftarrow$$

$$-y^{1/3} = 1 - x^{1/3}$$

$$y^{1/3} = -1 + x^{1/3}$$

$$y = (-1 + x^{1/3})^3$$

L)  $x^{1/3} - y^{1/3} = 1$

$$\frac{1}{3} x^{-2/3} - \frac{1}{3} y^{-2/3} \frac{dy}{dx} = 0$$

$$-\frac{1}{3} y^{-2/3} \frac{dy}{dx} = -\frac{1}{3} x^{-2/3}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{3} x^{-2/3}}{-\frac{1}{3} y^{-2/3}}$$

$$\boxed{\frac{dy}{dx} = \frac{y^{2/3}}{x^{2/3}}}$$

$$\frac{d^2y}{dx^2} = \frac{x^{2/3} \frac{2}{3} y^{-1/3} \frac{dy}{dx} - y^{2/3} \frac{2}{3} x^{-4/3}}{x^{4/3}}$$

$$= \frac{\cancel{2} x^{2/3} \left(\frac{y^{2/3}}{x^{2/3}}\right) - \cancel{2} y^{2/3}}{x^{4/3}}$$

$$\frac{\cancel{2} x^{1/3} \frac{2y^{1/3}}{3} - \frac{2y^{2/3}}{3x^{1/3}}}{x^{4/3} (3x^{1/3})}$$

$$\boxed{\frac{2x^{1/3} y^{1/3} - 2y^{2/3}}{3x^{5/3}}}$$