AP Problems Chapter 4

Mean Value Theorem

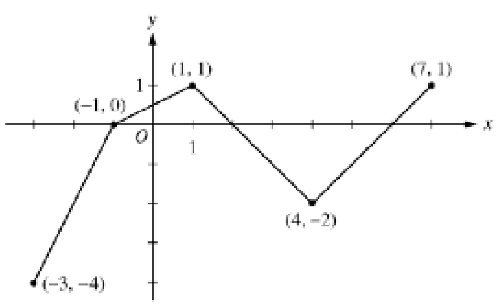
2013 BC3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

| t(minutes) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| C(t) ounces | 0 | 5.3 | 8.8 | 11.2 | 12.8 | 13.8 | 14.5 |

Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$. Justify your answer.

Let g be a continuous function with g(2) = 5. The graph of the piecewise-linear function $g'$, the derivative of g , is shown for $-3 \le x \le 7$.

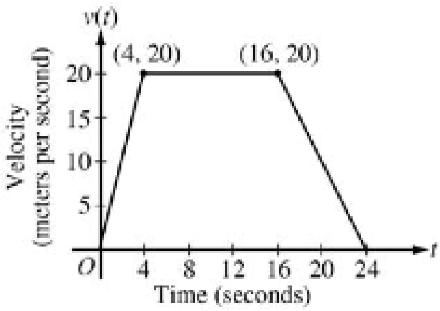


Graph of $g'$

Find the average rate of change of $g'(x)$, on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for -3 < c < 7 , such that $g''(c)$ is equal to this average rate of change? Why or why not?

2005 AB5

A car is traveling on a straight road. For $8 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph




Find the average rate of change of v over the interval $8 \le t \le 24$. Does the Mean Value guarantee a value of c, for 8 < c < 24, such that $v'(t)$ is equal to this average rate of change? Why of why not?

2004 BCB3

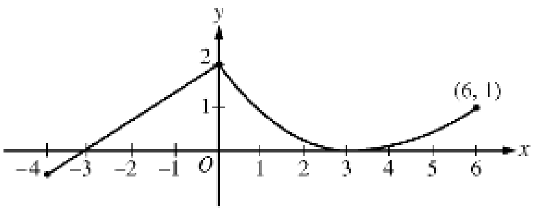
A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of v(t) are shown.

| t(min) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| v(t) (mpm) | 7 | 9.2 | 9.5 | 7 | 4.5 | 2.4 | 2.4 | 4.3 | 7.3 |

Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer

2009 BC3

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at x = 3, as shown in the figure above. On the interval 0 < x < 6, the function f is twice differentiable, with $f''(x) > 0$.



Graph of $f$

Is there a value a, , for which the Mean Value Theorem, applied to the interval [a, 6], guarantees a value c, a < c < 6, at which $f'(c) = \frac{1}{3}$? Justify your answer.

2011 BCB5

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position of the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values of B(t) and Ben's velocity, v(t), measured in meters per second, at selected times t.

| t(seconds) | 0 | 10 | 40 | 60 |
|---|---|---|---|---|
| B(t) (meters) | 100 | 136 | 9 | 49 |
| V(t) meters per second | 2 | 2.3 | 2.5 | 4.6 |

For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| f(x) | 2 | 3 | 4 | 3 | 2 |

83. The function f is continuous and differentiable on the closed interval [0, 4]. The table above gives selected values of f on this interval. Which of the following statements must be true?

A) The minimum value of f on [0, 4] is 2.
B) The maximum value of f on [0, 4] is 4
C) f(x) > 0 for 0 < x < 4
D) $f'(x) < 0$ for 2 < x < 4
E) There exists c, with 0 < c < 4, for which $f'(c) = 0$

92. Let f be the function defined by f(x) = x + ln(x). What is the value of c for which the instantaneous rate of change of f at x = c is the same as the average rate of change of f over [1, 4]?

A) 0.456 B) 1.244 C) 2.164 D) 2.342 E) 2.452

6. (calculator not allowed)

If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number $c$ in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be $c$?

(A) $\frac{2\pi}{3}$
(B) $\frac{3\pi}{4}$
(C) $\frac{5\pi}{6}$
(D) $\pi$
(E) $\frac{3\pi}{2}$

3. (calculator not allowed)

Let $f$ be the function given by $f(x) = x^3 - 3x^2$. What are all values of $c$ that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 3]$?

(A) 0 only
(B) 2 only
(C) 3 only
(D) 0 and 3
(E) 2 and 3

8. (calculator not allowed)

The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between (0, 0) and (4, 2). What are the coordinates of this point?

(A) (2, 1)
(B) (1, 1)
(C) $(2, \sqrt{2})$
(D) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
(E) None of the above