Pre-Calculus Practice Test C (Sums/Pascals Triangle/Partial Fractions)

Prove the sum by using the Sums of Powers of Integers Formulas. SHOW ALL YOUR WORK.

1. $\sum_{n=1}^{20} n^5 = \dfrac{20^2(20+1)^2(2(20)^2+2(20)-1)}{12}$

$= 12{,}333{,}300$

2. $\sum_{n=1}^{30} 3n^4 - 5n^3$

$3\left(\dfrac{30(30+1)(2(30)+1)(3(30)^2+3(30)-1)}{30}\right) - 5\left(\dfrac{30^2(30+1)^2}{4}\right)$

$1582 19970 - 1081125$

$147{,}40872$

Determine the limit of the following sequences. Use limit notation to show your answer.

3. $a_n = \dfrac{6n^3+5n-3}{3n^5-3n+2}$

$\lim_{n\to\infty} a_n = 0$

4. $a_n = \dfrac{7n^2+5n-3}{5n^2-3n+2}$

$\lim_{n\to\infty} a_n = \dfrac{7}{5}$

5. $a_n = \dfrac{8n-3}{6n^2-3n+2}$

$\lim_{n\to\infty} a_n = 0$

6. $a_n = \left(\dfrac{7n^3+4n-1}{2n^3-5n+2}\right)\left(\dfrac{2n^4+8n-3}{5n^4-6n+2}\right)$

$\left(\dfrac{7}{2}\right)\left(\dfrac{2}{5}\right) = \dfrac{7}{5}$

$\lim_{n\to\infty} a_n = \dfrac{7}{5}$

7. $a_n = \dfrac{9n^3+8n-1}{3n^5-7n+2} + \dfrac{8n^5+9n-1}{9n^5-5n+2}$

$0 + \dfrac{8}{9} = \dfrac{8}{9}$

$\lim_{n\to\infty} a_n = \dfrac{8}{9}$

8. Use the following series to complete part a and b.

$\sum_{n=1}^{\infty} \dfrac{(-5)^{n-1}}{n} x^n$

$\dfrac{(-5)^{1-1}}{1} x^1 \quad \dfrac{(-5)^{2-1}}{2} x^2 \quad \dfrac{(-5)^{3-1}}{3} x^3 \quad \dfrac{(-5)^{4-1}}{4} x^4 \quad \dfrac{(-5)^{5-1}}{5} x^5$

a. Write the First 6 Terms of the following series.

$x - \dfrac{5}{2}x^2 + \dfrac{25}{3}x^3 - \dfrac{125}{4}x^4 + 125x^5 - \dfrac{3125}{6}x^6$

b. Use the answer from part (a) to substitute x^2 into the series. Simplify your answer.

$x^2 - \dfrac{5}{2}x^4 + \dfrac{25}{3}x^6 - \dfrac{125}{4}x^6 + 125x^{10} - \dfrac{3125}{6}x^{12}$

Use Pascals Triangle to expand the binomial.

9. $(3x - 4y)^6$

$1(3x)^6 \quad 6(3x)^5(-4y) \quad 15(3x)^4(-4y)^2 \quad 20(3x^3)(-4y)^3 \quad 15(3x^2)(-4y)^4 \quad 6(3x)(-4y)^5 \quad 1(-4y)^6$

$729x^6 - 5832x^5y + 19440x^4y^2 - 34560x^3y^3 + 34560x^2y^4 - 18432xy^5 + 4096y^6$

10. $(5x + 3y)^7$

$\cancel{1(5x)^7 \quad 7(5x)^6 \quad 21(5x)^5 \quad 35(5x)^4 \quad 21(5x)^3 \quad 7(5x \quad 1}$

$1(5x)^7 \quad 7(5x)^6(3y) \quad 21(5x)^5(3y)^2 \quad 35(5x)^4(3y)^3 \quad 35(5x)^3(3y)^4 \quad 21(5x)^2(3y)^5 \quad 7(5x)(3y)^6 \quad 1(3y)^7$

$78125x^7 + 328125x^6y + 590625x^5y^2 + 590625x^4y^3 + 354375x^3y^4 + 127575x^2y^5$

$+ 25515xy^6 + 2187y^7$

Find the coefficient of the given term in the binomial expansion

11. Term: x^4 of $(2x + 5)^8$

$70(2x)^4(5)^4$

700,000

12. Term: x^3y^6 of $(3x + 2y)^9$

$64(3x)^3(2y)^6$

110592

Find the partial fraction decomposition.

13. $\frac{5}{x^2-8x} = \frac{A}{x} + \frac{B}{x-8} \qquad = \frac{-5}{8x} + \frac{5}{8(x-8)}$

$5 = A(x-8) + Bx$

Let $x=8$

$5 = 8B$

$B = \frac{5}{8}$

Let $x=0$

$5 = -8A$

$A = -\frac{5}{8}$

14. $\frac{x-4}{x^2+9x+8} = \frac{A}{x+8} + \frac{B}{x+1} \qquad \frac{12}{7(x+8)} - \frac{5}{7(x+1)}$

$x-4 = A(x+1) + B(x+8)$

Let $x=-1$

$-1-4 = 7B$

$-5 = 7B$

$B = \frac{-5}{7}$

Let $x=-8$

$-8-4 = -7A$

$-12 = \cancel{A} -7A$

$A = \frac{12}{7}$

15. $\frac{3x-2}{x^2-25} = \frac{A}{x-5} + \frac{B}{x+5} \qquad \frac{13}{10(x-5)} + \frac{17}{10(x+5)}$

$3x-2 = A(x+5) + B(x-5)$

Let $x=-5$

$3(-5)-2 = -10B$

$-17 = -10B$

$\frac{17}{10} = B$

Let $x=5$

$3(5)-2 = 10A$

$13 = 10A$

$\frac{13}{10} = A$