

What you will learn about:
Higher Roots

The Nth Root of A Number

If $b^n = a$, is the *nth root of a number a*.

The principal nth root of a is written $\sqrt[n]{a}$.

n is called the **index** of the radical.

$$4^3 = 64$$

$$3^4 = 81$$

$$(-2)^5 = -32$$

$$\sqrt[3]{64} = 4$$

$$\sqrt[4]{81} = 3$$

$$\sqrt[5]{-32} = -2$$

PROPERTIES OF $\sqrt[n]{a}$

When n is an even number and

- $a \geq 0$, then $\sqrt[n]{a}$ is a real number
- $a < 0$, then $\sqrt[n]{a}$ is not a real number

When n is an odd number, $\sqrt[n]{a}$ is a real number for all values of a .

Simplify:

$$(3)^4 = 81$$

$$\sqrt[3]{8} = 2$$

$$\sqrt[4]{81} = 3$$

$$\sqrt[5]{32} = 2$$

$$\sqrt[3]{27} = 3$$

$$\sqrt[4]{256} = 4$$

$$\sqrt[5]{243} = 3$$

$$\sqrt[n]{a}$$

$\sqrt{\quad}$ \rightarrow radical

a \rightarrow radicand

n \rightarrow index

$$2^3 = 8$$

$$\sqrt[3]{8} = 2$$

$$\sqrt[3]{1000}$$

10

$$\sqrt[4]{16}$$

2

$$\sqrt[5]{32}$$

2

$$\sqrt[3]{-125}$$

-5

$$\sqrt[4]{-16}$$

No Solution

$$\sqrt[5]{-243}$$

-3

$$\sqrt[3]{-216}$$

-6

$$\sqrt[4]{-81}$$

No Real #

$$\sqrt[5]{-1024}$$

-4

SIMPLIFYING ODD AND EVEN ROOTS

For any integer $n \geq 2$,

when n is odd $\sqrt[n]{a^n} = a$
when n is even $\sqrt[n]{a^n} = |a|$

We must use the absolute value signs when we take an even root of an expression with a variable in the radical.

Simplify:

$$\sqrt{x^2} = |x|$$

$$\sqrt[3]{n^3} = n$$

$$\sqrt[4]{p^4} = |p|$$

$$\sqrt[5]{y^5} = y$$

$$\sqrt[3]{y^{18}} = y^6$$

$$\sqrt[4]{z^4} = |z|$$

$$\sqrt[5]{c^{20}} = c^4$$

$$\sqrt[6]{d^{24}} = |d^4|$$

$$\sqrt[3]{27x^{27}}$$

$3x^9$

$$\sqrt[5]{243q^{25}}$$

$3q^5$

$$\sqrt[4]{81x^{28}}$$

$|3x^7|$

$$\sqrt[3]{125p^9}$$

$5p^3$

$$\sqrt[3]{x^4} = \sqrt[3]{x^3} \cdot \sqrt[3]{x}$$

$$\frac{\sqrt[3]{x^4}}{x \sqrt[3]{x}}$$

$$\frac{\sqrt[4]{x^7}}{\sqrt{x^4} \cdot \sqrt[4]{x^3}} = \frac{|x| \sqrt[4]{x^3}}{|x| \sqrt[4]{x^3}}$$

$$\frac{\sqrt[5]{p^8}}{\sqrt{p^5} \cdot \sqrt[5]{p^3}} = \frac{p \sqrt[5]{p^3}}{p \sqrt[5]{p^3}}$$

$$\frac{\sqrt[6]{t^{13}}}{\sqrt[6]{t^{12}} \cdot \sqrt[6]{t}} = \frac{t^2 \sqrt[6]{t}}{t^2 \sqrt[6]{t}}$$

$$\frac{\sqrt[3]{16}}{\sqrt[3]{8} \cdot \sqrt[3]{2}} = \frac{2 \sqrt[3]{2}}{2 \sqrt[3]{2}}$$

$$\frac{\sqrt[4]{243}}{\sqrt[4]{81} \cdot \sqrt[4]{3}} = \frac{13 \sqrt[4]{3}}{13 \sqrt[4]{3}}$$

$$\frac{\sqrt[3]{625}}{\sqrt[3]{125} \cdot \sqrt[3]{5}} = \frac{5 \sqrt[3]{5}}{5 \sqrt[3]{5}}$$

$$\frac{\sqrt[4]{729}}{\sqrt[4]{81} \cdot \sqrt[4]{9}} = \frac{13 \sqrt[4]{9}}{13 \sqrt[4]{9}}$$

$$\sqrt[3]{24} \cdot \sqrt[3]{x^7}$$

$$\sqrt[3]{8} \cdot \sqrt[3]{3} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{x} = 2 \sqrt[3]{3} \cdot x^2 \sqrt[3]{x}$$

$$\sqrt[3]{24x^7} = (2x^2) \sqrt[3]{3x}$$

$$\frac{\sqrt[4]{80y^{14}}}{\sqrt[4]{16} \cdot \sqrt[4]{5} \cdot \sqrt[4]{y^{12}} \cdot \sqrt[4]{y^2}} = \frac{|2y^3| \sqrt[4]{5y^2}}{|2y^3| \sqrt[4]{5y^2}}$$

$$\sqrt[3]{54p^{10}}$$

$$\sqrt[4]{162n^7}$$

$$13n \sqrt[4]{2n^3}$$

$$\sqrt[3]{-27}$$

$$\sqrt[3]{-108}$$

$$\sqrt[3]{-625}$$

$$\sqrt[4]{\frac{x^7}{x^3}} = \sqrt[4]{x^4} = |x|$$

$$\sqrt[3]{\frac{m^{13}}{m^7}} = \sqrt[3]{m^6} = m^2$$

$$\sqrt[4]{\frac{y^{17}}{y^5}} = \sqrt[4]{y^{12}} = |y^3|$$

$$\frac{\sqrt[3]{-108}}{\sqrt[3]{2}}$$

$$\sqrt[3]{-54}$$

$$\sqrt[3]{-27} \cdot \sqrt[3]{2} = -3 \sqrt[3]{2}$$

$$\frac{\sqrt[4]{96x^7}}{\sqrt[4]{3x^2}}$$

$$\sqrt[4]{32x^5}$$

$$\frac{\sqrt[4]{16} \cdot \sqrt[4]{2}}{|2x| \sqrt[4]{2x}}$$

$$\frac{\sqrt[3]{-192}}{\sqrt[3]{3}} = \sqrt[3]{-64}$$

$$= -4$$

$$\sqrt[3]{\frac{24x^7}{y^3}}$$

$$\sqrt[4]{\frac{48x^{10}}{y^8}}$$

$$\sqrt[3]{\frac{108c^{10}}{d^6}}$$

Simplify:

$$\sqrt[3]{4x} + 4\sqrt[3]{4x}$$

$5\sqrt[3]{4x}$

$$4\sqrt[4]{8} - 2\sqrt[4]{8}$$

$$\sqrt[3]{192} - \sqrt[3]{81}$$

$$\sqrt[4]{48} + \sqrt[4]{243}$$

$$\sqrt[3]{24x^4} - \sqrt[3]{-81x^7}$$

$$\sqrt[4]{243r^{11}} + \sqrt[4]{768r^{10}}$$

What you will learn about:
Rational Exponents

Rational Exponent $a^{\frac{1}{n}}$

If $\sqrt[n]{a}$ is a real number $n \geq 2$, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

Write as a radical expression:

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$y^{\frac{1}{3}} = \sqrt[3]{y}$$

$$z^{\frac{1}{4}} = \sqrt[4]{z}$$

Write as rational exponents:

$$\sqrt{t} = t^{\frac{1}{2}}$$

$$\sqrt[3]{h} = h^{\frac{1}{3}}$$

$$\sqrt[4]{m} = m^{\frac{1}{4}}$$

Simplify:

$$25^{\frac{1}{2}} = \sqrt{25} \\ = 5$$

$$8^{\frac{1}{3}} = \sqrt[3]{8} \\ = 2$$

$$16^{\frac{1}{4}}$$

$$(-64)^{\frac{1}{3}}$$

$$-64^{\frac{1}{3}}$$

$$(64)^{-\frac{1}{3}}$$

$$\sqrt{x^4} \\ = (x^4)^{\frac{1}{2}}$$

$$(-16)^{\frac{1}{4}}$$

$$-16^{\frac{1}{4}}$$

$$(16)^{-\frac{1}{4}}$$

Simplify Expressions with $a^{\frac{m}{n}}$.

$a^{\frac{m}{n}}$ → Power
→ root

Write with rational exponents:

$$\sqrt{x^5} = x^{\frac{5}{2}}$$

$$\sqrt[4]{z^3} = z^{\frac{3}{4}}$$

$$\sqrt[6]{p^7} = p^{\frac{7}{6}}$$

Simplify:

$$9^{\frac{3}{2}} = (\sqrt{9})^3 \\ = 27$$

$$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 \\ = 9$$

$$625^{\frac{3}{4}}$$

$$16^{-\frac{3}{2}}$$

$$32^{-\frac{2}{5}}$$

$$8^{-\frac{5}{3}}$$

Simplify

$$2^{\frac{1}{2}} \cdot 2^{\frac{5}{2}}$$

$$x^{\frac{2}{3}} \cdot x^{\frac{4}{3}}$$

$$z^{\frac{3}{4}} \cdot z^{\frac{5}{4}}$$

$$(x^4)^{\frac{1}{2}}$$

$$(y^6)^{\frac{1}{3}}$$

$$\left(z^{\frac{2}{3}}\right)^9$$

$$\frac{u^{\frac{5}{4}}}{u^{\frac{1}{4}}}$$

$$\frac{v^{\frac{3}{5}}}{v^{\frac{2}{5}}}$$

$$\frac{x^{\frac{2}{3}}}{x^{\frac{5}{3}}}$$

$$\left(27u^{\frac{1}{2}}\right)^{\frac{2}{3}}$$

$$\left(8v^{\frac{1}{4}}\right)^{\frac{2}{3}}$$

$$\left(32x^{\frac{1}{3}}\right)^{\frac{3}{5}}$$

$$\left(81n^{\frac{2}{5}}\right)^{\frac{3}{2}}$$

$$(m^3n^9)^{\frac{1}{3}}$$

$$(p^4q^8)^{\frac{1}{4}}$$

$$\frac{x^{\frac{3}{4}} \cdot x^{-\frac{1}{4}}}{x^{-\frac{6}{4}}}$$

$$\frac{y^{\frac{4}{3}} \cdot y^{\frac{2}{3}}}{y^{-\frac{2}{3}}}$$

$$\frac{m^{\frac{2}{3}} \cdot m^{-\frac{1}{3}}}{m^{-\frac{5}{3}}}$$

$$\frac{u^{\frac{4}{5}} \cdot u^{-\frac{2}{5}}}{u^{-\frac{13}{5}}}$$