What you will learn about:
Solving Rational Expressions

$x \quad x \quad 3$
$3 \quad 3 \quad 6$
$6 \quad 2 \quad 3$
$2 \cdot x \cdot 3$

$x - x = 6x$
$3 \quad 3$
$6 \quad 2 \cdot 3$

Extraneous Solution

An algebraic solution that would cause any of the expressions in the original equation to be undefined.

Solve equations with rational expressions

Step 1 – Note any value of the variable that would make the denominator zero.

Step 2 – Find the least common denominator of all denominators in the equation.

Step 3 – Clear the fractions by multiplying both sides of the equation by the LCD.

Step 4 – Solve the resulting equation.

Step 5 – Check
If any values found in step 1 are algebraic solutions, discard them.
Check any remaining solutions in the original equation

$x \neq 0$

Solve: $\left(\frac{1}{x}\right)+\left(\frac{1}{3}\right)=\left(\frac{5}{6}\right)$ (6x)

$6+2x=5x$
$-2x \quad -2x$
$6=3x$
$x=2$ ✓

$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

$y \neq 0$

Solve: $\left(1\right)-\left(\frac{5}{y}\right)=\left(\frac{6}{y^2}\right)$ (y²) LCD = y^2

$y^2-5y=-6$
$y^2-5y+6=0$
$(y-2)(y-3)=0$
$y=2 \quad y=3$

LCD = $(3u-2)(2u)$

Solve: $\left(\frac{5}{3u-2}\right)=\left(\frac{3}{2u}\right)$ $(3u-2)(2u)$

$3u-2=0$ → $u \neq \frac{2}{3}$
$2u=0$ → $u \neq 0$

$2u(5)=3(3u-2)$
$10u=9u-6$
$u=-6$

$y \neq 0$ LCD = $15y$

Solve: $\left(\frac{1}{x}\right)+\left(\frac{2}{3}\right)=\left(\frac{1}{5}\right)$ $15x$ $15y$ $15y$

$15+10y=3y$
$-10y \quad -10y$
$15=-7y$
$y=-\frac{15}{7}$ ✓

$b \neq 0$
LCD = b^2

Solve: $\left(1\right)-\left(\frac{4}{b}\right)=\left(\frac{12}{b^2}\right)$ b^2 b^2 b^2

$b^2-4b=12$
$b^2-4b-12=0$
$(b-6)(b+2)=0$
$b=6 \quad b=-2$

LCD = $(5n+1)(3n)$

Solve: $\left(\frac{3}{5n+1}\right)=\left(\frac{2}{3n}\right)$ $(5n+1)(3n)$ $(5n+1)(3n)$

$5n+1=0$ → $n \neq -\frac{1}{5}$
$3n=0$ → $n \neq 0$

$3(3n)=2(5n+1)$
$9n=10n+2$
$-n=2$
$n=-2$

$p \neq \pm 2$

LCD $= (p-2)(p+2)$

Solve: $\frac{2}{p+2} + \frac{4}{p-2} = \frac{p-1}{p^2-4}$.

$2(p-2) + 4(p+2) = p-1$

$2p-4+4p+8 = p-1$

$6p+4 = p-1$

$5p+4 = -1$

$5p = -5 \quad p = -1$

$\frac{2}{-1+2} + \frac{4}{-1-2} = \frac{-2}{1-4}$

$2 + \frac{4}{-3} \stackrel{?}{=} \frac{-2}{-3}$

$\frac{-6}{-3} + \frac{4}{-3} = \frac{-2}{-3}$

$q \neq 4 \quad q \neq 3$

LCD $= (q-4)(q-3)$

Solve: $\frac{4}{q-4} - \frac{3}{q-3} = 1.$

$4(q-3) - 3(q-4) = (q-4)(q-3)$

$4q-12-3q+12 = q^2-7q+12$

$q = q^2-7q+12$

$0 = q^2-8q+12$

$0 = (q-6)(q-2)$

$q = 6 \quad q = 2$

$m \neq 4, 1$

LCD $(m-4)(m-1)$

Solve: $\frac{m+11}{m^2-5m+4} = \frac{5}{m-4} - \frac{3}{m-1}.$

$m+11 = 5(m-1) - 3(m-4)$

$m+11 = 5m-5-3m+12$

$m+11 = 2m+7$

$11 = m+7$

m ~~$= 4$~~ No Solution

$n \neq 0$

LCD $= 12n$

Solve: $12n\left(\frac{n}{12}\right) + 12n\left(\frac{n+3}{3n}\right) = 12n\left(\frac{1}{n}\right)$

$n^2 + 4(n+3) = 12$

$n^2 + 4n + 12 = 12$

$n^2 + 4n = 0$

$n(n+4) = 0$

$n \neq 0 \quad n+4 = 0$

$n = -4$

$\frac{-1}{-12}$

$\frac{-4}{12} + \frac{-4+3}{-12} = -\frac{1}{4}$

$-\frac{4}{12} + \frac{1}{12}$

$-\frac{3}{12} = -\frac{1}{4}$

$y \neq \pm 6$

$LCD = (y-6)(y+6)$

$\frac{4}{10} = \frac{72}{-20} + 4$

$\frac{8}{20} = \frac{-72}{20} + \frac{80}{20}$

$\frac{8}{20} = \frac{8}{20}$ ✓

Solve: $\frac{y}{y+6} = \frac{72}{y^2-36} + 4.$

$y(y-6) = 72 + 4(y^2-36)$

$y^2 - 6y = 72 + 4y^2 - 144$

$y^2 - 6y = 4y^2 - 72$

$\frac{0}{3} = \frac{3y^2}{3} + \frac{6y}{3} - \frac{72}{3}$

$0 = y^2 + 2y - 24$

$0 = (y+6)(y-4)$

~~$y = -6$~~ $y = 4$

Solve: $\frac{x}{2x-2} - \frac{2}{3x+3} = \frac{5x^2-2x+9}{12x^2-12}.$

Solve: $\frac{z}{2z+8} - \frac{3}{4z-8} = \frac{3z^2-16z-16}{8z^2+16z-64}.$