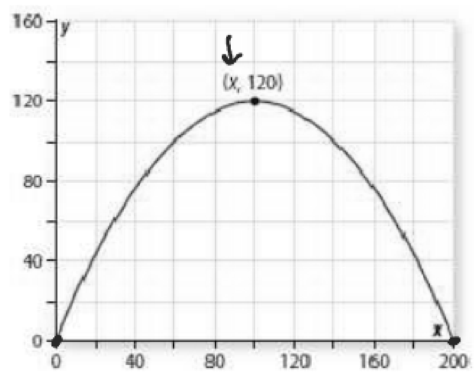
iii. What value(s) of x satisfy the equation $10 = h(x)$?

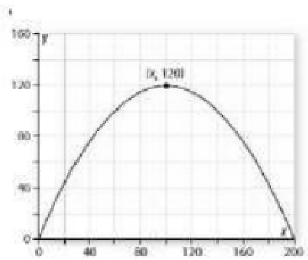
What do the value(s) tell about the flight of the player's shot?

Designing Parabolas

Like the airport structure, the Magic Moments restaurant was to be suspended about the ground by two giant parabolic arches-each 120 feet high and meeting the ground at point 200 feet apart. To prepare plans for the restaurant building, the designers had to develop and use functions whose graphs would match the planned arches. One way to tackle this design problem s to imagine a parabola drawn on a coordinate grid as shown below. Any parabola can be described as the graph of some quadratic function.



2. Using the idea from your earlier study of quadratic functions and their graphs, write the rule for a function with a parabolic graph contains the two points (0,0), (200,0) and a maximum point whose y-coordinate is 120. Use the hints in part a-d as needed.



a. The graph of the desired function has x-intercepts (0,0) and (200,0). How do you know that the graph of the function f(x)=x(x-200) has those same intercepts?

$f(x) = (x-0)(x-200)$

b. What is the x-coordinate of the maximum point on this graph? $x = 100$. halfway between x-intercepts

$y = a(x-p)(x-q)$

$(100, 120)$

$x \quad g(x)$

c. Suppose that g(x) has a rule in the form of $g(x) = [x(x-200)]$, for some particular value of k. What value of k will guarantee that g(x) = 120 at the maximum point of the graph.

$g(x) = a(x)(x-200)$

$g(x) = -.012x(x-200)$

d. Write the rule for g(x) in equivalent expanded form using the value of k you found in Part c.

$g(x) = a(x)(x-200)$

$120 = a(100)(100-200)$

$120 = a(100)(-100)$

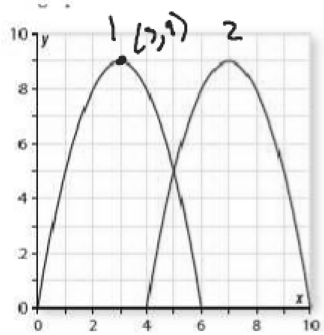
$g(x) = -.012x^2 - 2.4x$

$\frac{120}{-10,000} = \frac{-10,000a}{-10,000}$

$a = -.012$

$\frac{-3}{250}$

3. The logo chosen for Magic Moments continued the parabola theme with a large letter M drawn using two interesting parabolas. The idea is shown in the next graph.



$f(x) = a(x-0)(x-6)$

max $= (3,9)$

x f(x)

$9 = a(3)(3-6)$

$9 = a(3)(-3)$

$9 = -9a$

$a = -1$

a. Modify the strategy outlined in problem 2 to find a quadratic function that will produce the left most parabola in the M.

- Start with a function f(x) that has x-intercept at (0.0) and (6,0)
- Find the coordinates of the maximum point on the graph.
- Find the rule for a related function g(x) that has the same x-intercepts as f(x) but passes through the desired maximum point.
- Write the function rule for f(x) using an equivalent expanded form of the quadratic expression involved.

$f(x) = -x(x-6)$ $\quad f(x) = -x^2 + 6x$

g(x)

(4,0) (10,0)

$g(x) = a(x-4)(x-10)$

max (7,9)

$9 = a(7-4)(7-10)$

$9 = a(3)(-3)$

$9 = -9a$

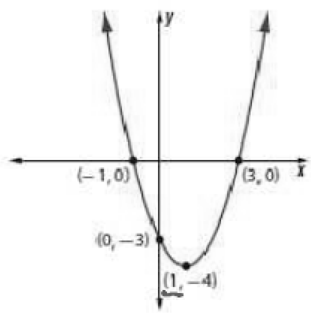
$a = -1$

b. Use a similar strategy to find a function g(x) that will produce the right most parabola.

$g(x) = -(x-4)(x-10)$ $\quad g(x) = -(x^2 - 14x + 40)$

$-x^2 + 14x - 40$

4. Explain why the next graph does or does not show the graph of f(x) = (x-3)(x+1)



5. Write rules for quadratic functions whose graphs have the following properties. If possible, write more than one function rule that meets the given conditions.

a. x-intercepts at (4,0) and (-1,0)

$f(x) = a(x-4)(x+1)$ / $f(x) = -a(x-4)(x+1)$

b. x-intercepts at (7,0) and (1,0) and graph opening upward

$g(x) = a(x-7)(x-1)$

c. x-intercepts at (7,0) and (1,0) and minimum point at (4, -10)

↑ ↑
x f(x)

$f(x) = a(x-7)(x-1)$

$-10 = a(4-7)(4-1)$

$-10 = a(-3)(3)$

$-10 = -9a$

$a = \frac{10}{9}$

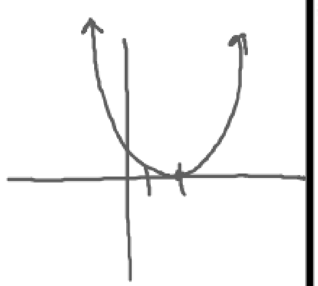
$f(x) = \frac{10}{9}(x-7)(x-1)$

	d. x-intercepts at (-5,0) and (0,0) and graph opens downward. $f(x) = -a(x+5)(x-0)$ $= -ax(x+5)$ e. x-intercepts (3,0) and (-5,0) and maximum point at (-1,8) $x \downarrow \nearrow f(x)$ $f(x) = a(x-3)(x+5)$ $8 = a(-1-3)(-1+5)$ $8 = a(-4)(4)$ $8 = a(-16)$ $-\frac{1}{2} = a$ $f(x) = -\frac{1}{2}(x-3)(x+5)$ f. x-intercepts at (3.5,0) and (0,0) and graph opens upward. g. x-intercepts at (4.5,0) and (1,0) and y-intercept at (0,9) $x \downarrow \nearrow f(x)$ $f(x) = a(x-4.5)(x-1)$ $9 = a(0-4.5)(0-1)$ $9 = a(-4.5)(-1)$ $9 = 4.5a$ $a = 2$ $f(x) = 2(x-4.5)(x-1)$ h. x-intercepts at (m,0) and (n,0)

i. only one x-intercept at (0,0)

x → f(x)

j. only one x-intercept at (2,0) and y-intercept at (0,6)



$f(x) = a(x-2)(x-2)$

$= \frac{3}{2}(x-2)(x-2)$

$f(x) = a(x-2)^2$

$6 = a(0-2)^2$

$6 = a(-2)^2$

$6 = 4a$

$a = \frac{3}{2}$

$f(x) = \frac{3}{2}(x-2)^2$