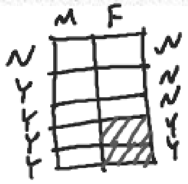
What you will learn about:
How to find P(A and B) when A and B are not independent

1. About half of all U.S. adults are male. The USA Today-reported data earlier indicated that three out of five adults sing in the shower. Some people think that males are more likely to sing in the shower than females. Suppose that they are right: 80% of males sing in the shower, but only 40% of females sing in the shower.

a. Make an area model that represents this situation.



b. Suppose you pick an adult at random. What it the probability that you get a female who sings in the shower?

$$\frac{2}{10} = \frac{1}{5}$$

c. Complete the following equation (in words) that describes how you found the probability in Part b.

$$\frac{1}{2} \cdot \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$$

P(*female and sings in the shower*) =

$P(\text{Female}) \cdot P(\text{Sings in Shower} \mid \text{Female})$

d. Suppose you pick and adult at random. What is the probability that you get a male who sings in the shower?

$$\frac{4}{10} = \frac{2}{5}$$

e. Write an equation in words that describes how you found the probability in Part d.

$P(\text{Male}) \cdot P(\text{Sings in Shower} \mid \text{male})$

$$\frac{1}{2} \cdot \frac{4}{5} = \frac{4}{10} = \frac{2}{5}$$

2. As you have seen in Problem 1, if events A and B are not independent, you can find the P(*A and B*) by using either one of the following rules.

P(*A and B*) = $P(A) \cdot P(B|A)$
P(*A and B*) = $P(B) \cdot P(A|B)$

a. For the situation of rolling a pair of dice once, let event A be rolling doubles and event B getting a sum of 8.

i. Using the sample space below, find each of the following probabilities.

A - Doubles
B - Sum of 8

Number on Second Die

Number on First Die

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

- P(A) $= \frac{1}{6}$
- P(B) $= \frac{5}{36}$
- $P(A|B) = \frac{1}{5}$
- $P(B|A) = \frac{1}{6}$
- P(A and B)

$P(A) = P(A|B)$

$\frac{1}{6} \neq \frac{1}{5}$

$P(A) \cdot P(B|A) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$

$P(B) \cdot P(A|B) = \frac{5}{36} \cdot \frac{1}{8} = \frac{1}{36} \leftarrow \frac{5}{180}$

ii. Verify that both rules for P(A and B) hold for the probabilities that you found in part i.

b. Show that both rules work for each of the following situations.

$P(\text{Doubles}) = P(\text{Doubles}|\text{sum } 7)$

$\frac{1}{6} \neq \frac{0}{6}$

Not Independent

i. You roll a pair of dice once. Event A is rolling doubles. Event B is getting a sum of 7.

$P(A) \cdot P(B|A)$

$\frac{1}{6} \cdot \frac{0}{6}$

0

$P(B) \cdot P(A|B)$

$\frac{1}{6} \cdot \frac{0}{6}$

0

$P(1 \text{ on } 1^{st}) = P(1 \text{ on } 1^{st} | \text{Sum } 7)$

$P(A) = \frac{1}{6}$

$P(B) = \frac{1}{6}$

$P(A|B) = \frac{1}{6}$

$P(B|A) = \frac{1}{6}$

ii. You roll a pair of dice once. Event A is getting 1 on the first die. Event B is getting a sum of 7.

$$P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$= P(B) \cdot P(A|B) = \frac{1}{6} = \frac{1}{6} = \frac{1}{36}$$

3. A Web site at Central Michigan University collects data from statistics students. In one activity, students were asked whether they were right-handed or left-handed. Students were also asked which thumb is on top when they fold their hands (intertwining their fingers). The following table shows the results for the first 80 students who submitted their information.

	Left-Handed	Right-Handed	Total
Left Thumb on Top	2	46	48
Right Thumb on Top	4	28	32
Total	6	74	80

Source: stat.cst.cmich.edu/statact/index.php

Suppose you pick one of these 80 students at random.

a. Find each probability.

i. P(left-handed) $\frac{6}{80} = \frac{3}{40}$

ii. P(left thumb on top) $= \frac{48}{80} = \frac{3}{5}$

iii. $P(left\ thumb\ on\ top | left-handed) = \frac{2}{6} = \frac{1}{3}$

iv. $\underline{P(left-handed | left\ thumb\ on\ top)} = \frac{2}{48} = \frac{1}{24}$

b. Are being left-handed and having the left thumb on top independent events? Are they mutually exclusive events?

$$P(\text{Left}) \overset{?}{=} P(\text{Left} | \text{Left thumb on top})$$

$$\frac{3}{40} \overset{?}{\neq} \frac{1}{24} \text{ Not Independent}$$

c. Use your results from Part a and the formula to find *P(left-handed and left thumb on top)*. Check your answer by using the table directly.

$P(\text{Left-Handed and left thumb on top}) =$

$P(\text{Left-Handed}) \cdot P(\text{Left thumb on top} | \text{Left handed})$

$$\frac{\cancel{3}}{40} \cdot \frac{1}{\cancel{3}} = \frac{3}{120} = \frac{1}{40}$$

4. Thin~~g~~k about a single roll of ~~say~~ two dice. For each of the situations below, tell whether the two events are mutually exclusive. Then tell whether they are independent.

a. Event A rolling doubles. Event B is getting a sum of 8.

Not mutually Exclusive (4,4)

$P(\text{Doubles}) = P(\text{Doubles} \mid \text{sum } 8)$

$\frac{1}{6} \neq \frac{1}{5}$ Not Independent

b. Event A is rolling doubles. Event B is getting a sum of 7.

Yes mutually Exclusive
No Doubles gives a sum of 7

$P(\text{Doubles}) = P(\text{Doubles} \mid \text{sum } 7)$

$\frac{1}{6} \neq \frac{0}{6}$ Not Independent

c. Event A is getting 1 on the first die. Event B is getting a sum 7.

No Roll of 1,6
will add to 7.

$P(A) = P(A \mid B)$

$\frac{1}{6} = \frac{1}{6}$ Independent Events

d. Event A is getting 1 on the first die. Event B is getting doubles.

$P(A) = P(A \mid D)$

$\frac{1}{6} = \frac{1}{6}$ Independent

5. Find the probability of the following.

a. P(left-handed [✓ or] left thumb on top)

$$P(LH) + P(LTT) - P(LH \text{ and } LTT)$$

$$\frac{6}{80} + \frac{48}{80} - \frac{2}{80} = \frac{52}{80} = \frac{13}{20}$$